TOTAL REFLECTION COEFFICIENTS OF LOW-ENERGY PHOTONS PRESENTED AS UNIVERSAL FUNCTIONS

by

Vladan LJUBENOV [1*], ***Rodoljub D. SIMOVIĆ*** [2, 3], ***Srpko MARKOVIĆ***, ***and Radovan D. ILIĆ*** [4]

[1]International Atomic Energy Agency, Vienna, Austria
[2] Faculty of Ecology and Environmental Protection, Belgrade, Serbia
[3] Public Company Nuclear Facilities of Serbia, Belgrade, Serbia
[4] Vinča Institute of Nuclear Sciences, Belgrade, Serbia

Scientific paper
UDC: 539.122:519.245
DOI: 10.2298/NTRP1002100L

The possibility of expressing the total particle and energy reflection coefficients of low-energy photons in the form of universal functions valid for different shielding materials is investigated in this paper. The analysis is based on the results of Monte Carlo simulations of photon reflection by using MCNP, FOTELP, and PENELOPE codes. The normal incidence of the narrow monoenergetic photon beam of the unit intensity and of initial energies from 20 keV up to 100 keV is considered, and particle and energy reflection coefficients from the plane homogenous targets of water, aluminum, and iron are determined and compared. The representations of albedo coefficients on the initial photon energy, on the probability of large-angle photon scattering, and on the mean number of photon scatterings are examined. It is found out that only the rescaled albedo coefficients dependent on the mean number of photon scatterings have the form of universal functions and these functions are determined by applying the least square method.

Key words: photon reflection, total number albedo, total energy albedo, Monte Carlo method, mean number of photon scatterings

INTRODUCTION

It has been known that particle reflection from a planar target can be described by universal functions and that has been the topic of extensive research, analyses, and application in the physics of ions – particularly in the analyses of the reflection of light low-energy ions from heavy targets (the energy domain of hundreds of eV to tens of keV) [1-5]. Recently, it has been shown that the reflection of high-energy (MeV domain) light ions can be described by the universal functions valid for all light ions and heavy targets as well [6]. Universal functions for the ion reflection have been found by rescaling, *i. e.*, replacing energy or reduced energy as an independent variable in the analyses by a new complex variable – function ν. This quantity denotes the number of large-angle scatterings of light ions in the target material during their penetration before their final stopping.

In this paper we investigate the possibility to describe photon reflection in the domain of initial energies of up to 100 keV, *i. e.*, to describe the total photon coefficients – the total number albedo $a_N(E_0, \theta_0)$ and the total energy albedo $a_E(E_0, \theta_0)$ – by universal functions valid for typical shielding materials used for protection from ionizing radiation. In order to explain clearly the baseline for the concept of universal functions, our discussion will start with the standard representation of the reflection coefficients dependent on the initial photon energy, then will proceed with the probability of large-angle photon scattering, and finally will focus on the mean number of photon scatterings before the final reflection from the planar target.

The results presented in this paper are a part of the systematic research of low-energy photon reflection performed during several years and partly published elsewhere [7-10]. Values for the total number and energy albedo were calculated based on the Monte Carlo simulations of photon reflection by MCNP [11], FOTELP [12], and PENELOPE [13] codes. The normal incidence of a photon beam, with the initial energies of

* Corresponding author; e-mail: V.Ljubenov@iaea.org

20 keV to 100 keV, to the homogenous infinite slab of a shielding material (water, aluminum, and iron) was modeled and simulated. Angular and energy dependent albedo coefficients were determined, as well as the mean number of consecutive scatterings before the final reflection from the target material. These data have been generated during a longer period from Monte Carlo simulations and from the subsequent analyses, and they have been used as a basis for the development of the universal function concept presented here.

DEFINITION OF REFLECTION COEFFICIENTS

Total albedo coefficients are defined as integrals of the differential particle albedo over the angle and energy [8, 9, 14]. Differential albedo is defined for a narrow photon beam of a unit intensity and initial energy E_0 which hits the planar half-space scattering media in the point selected as origin of the co-ordiante system under the polar angle θ_0, measured from the line perpendicular to the boundary plane. This quantity represents probability for the photon reflection from the unit area of the scattering media located arround the point of incidence (x, y, $z = 0$) where reflected photons have the energy in the unit interval dE arround the energy E and the direction in the unit solid angle dΩ arround the direction defined by the vector $\vec{\Omega}$. This differential albedo is denoted as $a(E_0,\theta_0;E,\theta,\varphi,x,y)$, where θ and φ are polar and azimuthal angle of the reflected photons.

The point of the photon detection in experimental practice is far enough from the boundary surface of the scattering media so it can be assumed that photons are reflected from the same point where the incident beam hits the media. Thus, differential albedo can be presented as a quantity which has no dependence on the space co-ordinates x and y

$$a(E_0,\theta_0;E,\theta,\varphi) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a(E_0,\theta_0;E,\theta,\varphi,x,y)\,\mathrm{d}x\,\mathrm{d}y \qquad (1)$$

and is named differential spectral albedo – the energy-angular distribution of backscattered radiation. Sometimes this quantity is called double-differential albedo in order to emphasize its differential character with regard to both energy E and the direction vector $\vec{\Omega}$.

Total albedo coefficients can be obtained by the integration of the double-differential albedo over the energy and solid angle domain: total number albedo

$$a_N(E_0,\theta_0) \int_0^{E_0}\mathrm{d}E\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi/2} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta\,\mathrm{d}\theta \qquad (2)$$

and total energy albedo

$$a_E(E_0,\theta_0) \frac{1}{E_0}\int_0^{E_0} E\,\mathrm{d}E\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi/2} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta\,\mathrm{d}\theta \qquad (3)$$

In practice, reflection coefficients are calculated based on the difference albedo coefficients $a_N^{ji}(E_0,\theta_0)$, generated from the Monte Carlo numerical experiment, which corresponds to the integrals of the differential spectral albedo $a(E_0,\theta_0;E,\theta,\varphi)$ over the selected photon energy interval E_j and the chosen solid angle segment Ω_i

$$a_N^{ji}(E_0,\theta_0) \int_0^{2\pi}\mathrm{d}\varphi\int_{\Delta E_j}\mathrm{d}E\int_{\Delta\theta_i} a(E_0,\theta_0;E,\theta,\varphi)\sin\theta\,\mathrm{d}\theta \qquad (4)$$

Difference number albedo $a_N^{ji}(E_0,\theta_0)$ represents probability for photon reflection from the boundary surface within the energy interval ΔE_j and the solid angle element $\Delta\Omega_i$ integrated over the azimuthal angle. Similar to the total number albedo, this quantity is dimensionless.

Details on the performed Monte Carlo simulations and on the way difference albedo coefficients $a_N^{ji}(E_0,\theta_0)$ for water, aluminum, and iron were calculated are presented in ref. [14]. The set of values for difference albedo coefficients $a_N^{ji}(E_0,\theta_0)$ calculated on the basis of the simulation results was obtained for nine equal intervals of the polar angle θ, each 10° wide, and for ten equally wide energy intervals (E_0/10 is the width of each energy group). The total number albedo can be obtained by summing difference coefficients $a_N^{ji}(E_0,\theta_0)$ over all the angle and energy intervals

$$a_N(E_0) \sum_{j=1}^{10}\sum_{i=1}^{9} a_N^{ji}(E_0,\theta_0) \qquad (5)$$

while total energy albedo is obtained according to definition (3)

$$a_E(E_0) \frac{1}{E_0}\sum_{j=1}^{10}\overline{E}_j\sum_{i=1}^{9} a_N^{ji}(E_0,\theta_0) \qquad (6)$$

where $\overline{E}_j$ denotes average energy of the j-th energy interval.

In the expressions (5) and (6) for the total reflection coefficients, argument θ_0 has been omitted as the analysis deals with the normal photon incidence ($\theta_0 = 0°$) to the target material only .

THREE REPRESENTATIONS OF THE REFLECTION COEFFICIENTS

Three materials were selected for the analyses – water, aluminum, and iron. Water represented a predominantly scattering media while iron was chosen as a predominantly absorbing media for photons. The considerations presented here cover the following representations of the reflection coefficients: (a) traditional representation – albedo dependence on the ini-

tial photon energy E_0, (b) novel representation – albedo dependence on the probability of large-angle photon scattering, *i. e.*, on parameter c , and (c) newest interpretation – the dependence of reflection coefficients on the mean number of photon scatterings $\bar{n}$.

Coefficients dependent on the initial photon energy E_0

The values for the total number albedo $a_N(E_0)$ for water, aluminum, and iron are given in tab. 1. For all three materials these coefficients were calculated by MCNP code for nine initial photon energies ranging from 20 keV to 100 keV, with the initial energy increment of 10 keV. The results were verified by FOTELP and PENELOPE codes for the initial photon energies of 40 keV, 60 keV, and 100 keV.

The agreement between the values calculated by MCNP and FOTELP codes for water is very good, ranging from less than 10% of relative discrepancy for the lower limit of the initial photon energies and reaching only 2-3% for the upper limit of 100 keV and the initial energies close to this one. PENELOPE code gives the values of the reflection coefficients laying between these two sets of results, a bit closer to the results obtained by FOTELP code. Similar conclusion about the agreement of the results is valid for aluminum, where all three sets of results are within 0.5-2% for the initial photon energy of 100 keV. For the iron target the discrepancies of the results are higher. MCNP and PENELOPE codes are in better agreement: for the initial photon energy of 40 keV relative discrepancies are bellow 30% while for 60 keV and 100 keV the agreement is much better reducing discrepancies to less than 2%. The FOTELP code results differ from the MCNP results more for low initial energies, but for 100 keV the agreement is within 4%. Generally, the results obtained by three different codes are closer for higher initial photon energies than for lower ones, and the satisfactory agreement was not obtained for the iron target and low initial photon energies. Such behavior of the results can be explained by high photon absorption in iron, especially for low initial photon energies (for E_0 = 40 keV the total number albedo for iron is more than 20 times lower than the albedo for water), so the simulation of the photon reflection has to be performed with very large number of photon histories to get reliable results. It has to be noted that the MCNP simulations involved three times more photon histories than the simulations with two other codes [8]. Thus, the FOTELP and PENELOPE results for the iron target and the lowest initial photon energies have the highest statistical uncertainties amongst the Monte Carlo results considered. For the initial photon energy of 100 keV, where nine times more photons are reflected from iron than for 40 keV, the reliability of the results is higher and the agreement between the three codes is quite satisfactory.

The results for the total energy albedo for water, aluminum and iron and the initial photon energies from 20 keV to 100 keV, with 10 keV energy increment, obtained from the simulations of photon reflection performed by MCNP code are presented in tab. 2.

The values of the reflection coefficients $a_N(E_0)$ and $a_E(E_0)$ for water, given there, are in a very good agreement with the referent results from Mashkovich Manual [15], which is illustrated in tab. 3.

Table 1. Total photon number albedo for water, aluminum, and iron obtained by numerical simulation using MCNP (1), FOTELP (2), and PENELOPE (3) codes

E_0 [keV]	Water			Aluminum			Iron		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
20	0.0478	0.0435		0.00728			0.0224		
30	0.128	0.117		0.0232			0.0102		
40	0.209	0.196	0.201	0.0490	0.0473	0.0517	0.00890	0.0053	0.0064
50	0.273	0.259		0.0809			0.0115		
60	0.319	0.306	0.309	0.114	0.110	0.115	0.0162	0.0142	0.0164
70	0.351	0.340		0.145			0.0221		
80	0.373	0.363		0.173			0.0291		
90	0.389	0.379		0.197			0.0365		
100	0.400	0.391	0.392	0.216	0.213	0.216	0.0444	0.0422	0.0451

Table 2. Total photon energy albedo for water, aluminum, and iron obtained by numerical simulation using MCNP code

E_0 [keV]	Water	Aluminum	Iron
20	0.0453	0.00686	0.00831
30	0.116	0.0215	0.00425
40	0.178	0.0425	0.00526
50	0.223	0.0686	0.00854
60	0.251	0.0953	0.0130
70	0.262	0.116	0.0175
80	0.266	0.133	0.0225
90	0.266	0.146	0.0277
100	0.264	0.156	0.0333

Table 3. Comparison of MCNP simulations with the referent results for total albedo coefficients

Albedo coefficient	Source of results	Initial photon energy E_0 [keV]		
		20	50	100
$a_N(E_0)$	MCNP simulation	0.048	0.273	0.400
	Mashkovich [15]	0.050	0.276	0.391
$a_E(E_0)$	MCNP simulation	0.045	0.224	0.264
	Mashkovich [15]	0.047	0.227	0.257

It can be concluded that for both analyzed total albedo coefficients there is a good agreement of the results calculated based on the MCNP simulations of photon reflection from water with the referent results for water. For the entire initial photon energy domain up to 100 keV discrepancies are within 2-3%.

Figures 1 and 2 show total reflection coefficients as a function of the initial photon energy E_0. It is obvious that both number and energy coefficients are not only dependent on the initial photon energy E_0, but there is also a strong dependency on the target material. That is why in many papers and manuals dealing with radiation protection the albedo coefficients for the limited number of the most important shielding materials have been presented in tables or graphs [15]. According to the best knowledge of the authors, there has been no attempt or example in the available literature to present total photon reflection coefficients as unique functions which would be valid for several important shielding materials.

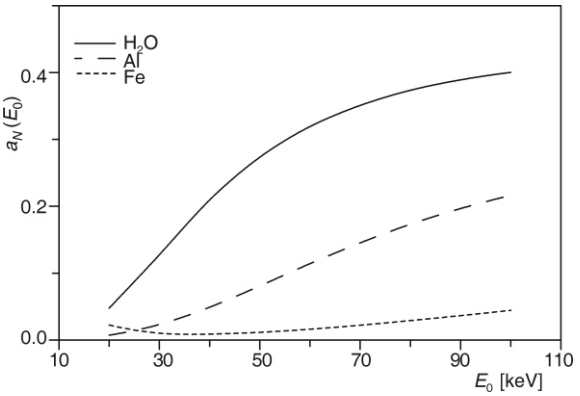

Figure 1. Total number albedo as a function of the initial photon energy E_0

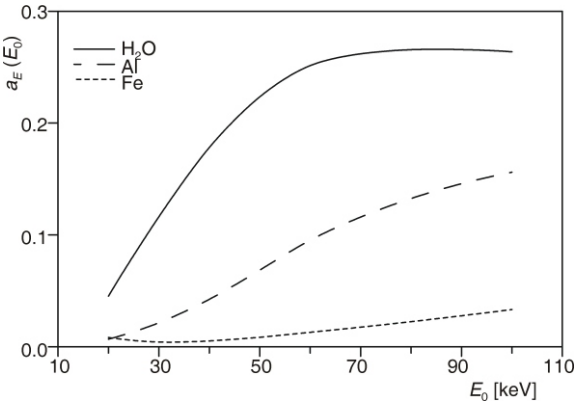

Figure 2. Total energy albedo as a function of the initial photon energy E_0

Coefficients dependent on the parameter c – probability of large-angle photon scattering

Probability for the photon scattering in the energy domain of up to 100 keV is determined by the ratio of the sum of linear interaction coefficients for coherent (Rayleigh) scattering μ_{coh} and non-coherent (Compton) scattering μ_C and the total interaction coefficient μ (the sum of the linear coefficients for photo-electric effect, coherent and non-coherent scattering). As there are small-angle photon scatterings during a coherent interaction, where the reduced energy of the scattered photon is very close to the energy of the incident photon, for not so precise analyses of the photon interactions it is possible to neglect the contribution of the coherent photon scatterings. Then the probability of photon scattering is defined by [10]

$$c \quad \frac{\mu_C}{\mu} \tag{7}$$

where μ is the sum of the coefficients for photo-electric effect and Compton scattering. The parameter c can be understood as a probability of large-angle photon scattering.

Recently, by introducing this parameter into photon reflection analyses, a simpler and more clear insight into the photon reflection process in the energy domain of up to 100 keV has been achieved [10, 14]. It has been demonstrated that with increasing parameter c the number of scatterings that photons undergo before the final escape from the material increases as well and that value c determines the angular and energy distribution of the albedo coefficients. Here the behavior of the total albedo coefficients as the functions of the parameter c is analysed in brief. It has to be noted that with the increase of the initial photon energy up to 100 keV the parameter c reaches the value close to 1 for water or approximately 0.9 for aluminum, while for iron the initial photon energy of 100 keV corresponds to c value of about 0.4 (tab. 4).

Figures 3 and 4 show total albedo coefficients as functions of the parameter c instead of the initial photon energy E_0. This rescaling of the abscissa (x-axis) was done by calculating c value for each material for the given initial photon energy E_0 based on the data from literature [15].

It can be seen from figs. 3 and 4 that total reflection coefficients for different materials do not differ too much for the same c values and that their graphs

Table 4. Parameter c' – probability of large-angle photon scattering

Material	Initial photon energy E_0 [keV]				
	20	40	60	80	100
Water	0.245	0.763	0.922	0.968	0.983
Aluminum	0.042	0.298	0.607	0.791	0.885
Iron	0.005	0.039	0.122	0.247	0.389

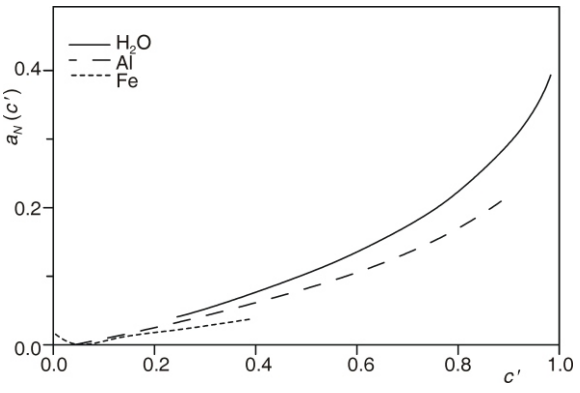

Figure 3. Total number albedo as a function of the probability of large-angle photon scattering *c*

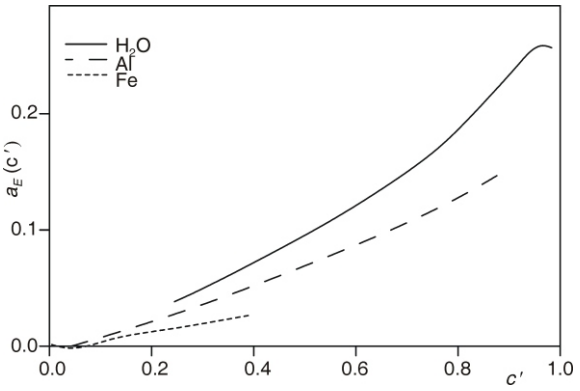

Figure 4. Total energy albedo as a function of the probability of large-angle photon scattering *c*

have similar increasing trends. Although rescaling the x-axis from E_0 to c did not result in obtaining universal functions for total photon reflection, this step contributed in the right direction anyway – towards the better compliance of the reflection coefficients for different materials.

Coefficients dependent on the parameter $\bar{n}$ – mean number of photon scatterings

The dependence of the total reflection coefficients on the mean number of photon scatterings $\bar{n}$ before the final escape from the target was analysed based on the results of Monte Carlo simulations performed by FOTELP code. For all three shielding materials and for each selected initial photon energy E_0, the mean number of photon scattering $\bar{n}$ before escaping from the target media was determined by the numerical processing of the simulation results. The outcomes of the FOTELP code simulations were the angular-energy distributions of reflected photons decomposed according to the number of photon collisions in the material and $\bar{n}$ was calculated from the data corresponding to the absolute peak of these distributions (see tab. 5). This peak always belongs to the exit polar angle interval θ (40°, 50°), where θ is measured from the outward normal of the incident target plane. Looking at the exit photon energy, the peak of the reflected photon energy distribution is shifted from higher to lower energies with the increase in number of scatterings undergone before the final photon reflection. It was already noted [10] that the number of photon scatterings and, consequently, the mean number of scatterings $\bar{n}$ depended on both the target material and the initial photon energy E_0: in lighter materials and for higher energies, photons are reflected after a larger number of scatterings than in heavier materials and at lower initial photon energies. This is how figs. 5 and 6 were generated.

Table 5. Parameter $\bar{n}$ – mean number of photon scatterings before escaping target material

Material	Initial photon energy E_0 [keV]				
	20	40	50	60	100
Water	1.24	–	1.93	–	2.63
Aluminum	–	1.29	–	1.43	1.69
Iron	–	1.06	–	–	1.23

It can be seen that the values for total reflection coefficients form only one curve with the common shape for all the materials considered. Universal curves in the form of the second degree polynomials were determined by applying the least square methods: for total number albedo

$$a_N(\bar{n}) \quad 0.639 \quad 0.687\,\bar{n} \quad 0.112\,\bar{n}^2 \qquad (8)$$

and for total energy albedo

$$a_E(\bar{n}) \quad 0.604 \quad 0.680\,\bar{n} \quad 0.133\,\bar{n}^2 \qquad (9)$$

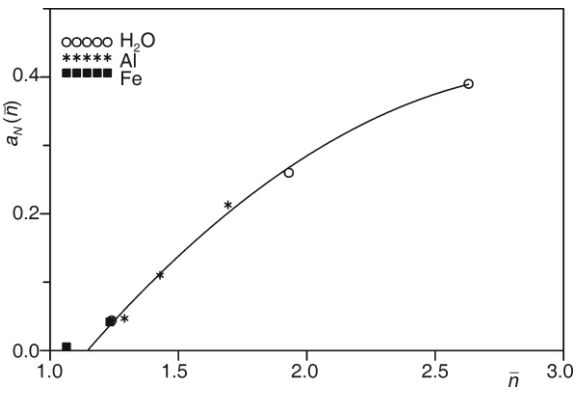

Figure 5. Total number albedo as a function of the mean number of photon scatterings $\bar{n}$

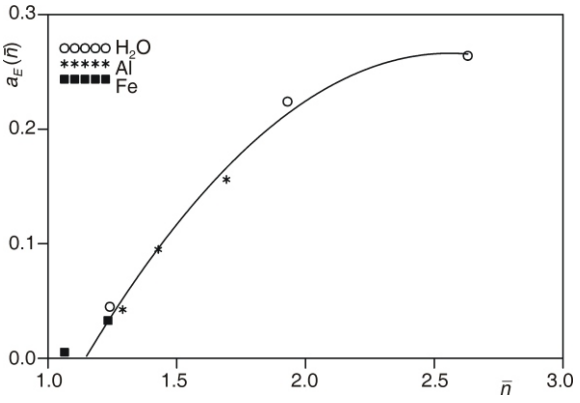

Figure 6. Total energy albedo as a function of the mean number of photon scatterings $\bar{n}$

CONCLUDING CONSIDERATIONS

This paper demonstrates that total albedo coefficients for photon reflection in energy domain from 20 keV to 100 keV can be described by universal functions. The scaling parameter in photon reflection is $\bar{n}$ – the mean number of consecutive photon scatterings undergone before escaping from the target. This is the main difference between the photon reflection and ion reflection where scaling parameter ν represents the number of ion collisions with large-angle scattering before the final stopping in the target material. Possibility to define reflection coefficients as universal functions which are independent from the target material emphasizes the domination of the statistical nature of the reflection process over the characteristics of a single photon interaction in the material.

Universal reflection functions were formulated for the normal photon incidence on the planar target. Here, we did not treat the behaviour of photon beams penetrating into the target under oblique angles. In such a case, the contribution of once scattered photons to total reflection is higher, while the contribution of photons scattered more than once decreases. Still the generalization of conclusions presented in this paper to the more universal case of oblique photon incidence seems possible, but not well-founded without adequate experimental proofs or Monte Carlo simulation results.

Based on the results of photon reflection analyses published earlier [16], the extension of the energy domain towards higher energies for which universal functions might be valid could be expected, but the results of systematic Monte Carlo simulations are necessary to support such conclusion.

ACKNOWLEDGEMENT

This work is supported by the Ministry of Science and Technological Development of the Republic of Serbia under the contract 141046.

REFERENCES

[1] Tilinin, I. S., Reflection of Low Energy Atomic Particles from Solid Surface, *Phys. Chem. Mech. Surf., 2* (1984), pp. 640-660
[2] Vicanek, M., Urbassek, N. M., Reflection Coefficient of Low-Energy Light Ions, *Phys. Rev. B, 44* (1991), 14, pp. 7234-7242
[3] Luo, Z., Hou, Q., Can the Light Ion Transport in Heavy Targets be Characterized by a Single Parameter?, *Nucl. Instr. and Meth. B, 95* (1995), 1, pp. 1-6
[4] Vukanić, J., Simović, R., Analytical Solutions of the Boltzmann Transport Equation for Light Ion Reflection, *Fus. Technol., 30* (1996), 2, part 1, pp. 372-382
[5] Simović, R., Vukanić, J., The Albedo Problem of Low-Energy Light Ions Treated Analytically in the DP0 Flux Approximation, *J. Phys. D: Appl. Phys., 30* (1997), 16, pp. 2331-2337
[6] Vukanić, J., Simović, R., Scaling Properties in Single Collision Model of Light Ion Reflection, *Nucl. Instr. and Meth. B, 215* (2004), 3-4, pp. 337-344
[7] Marković, S., Ljubenov, V., Ciraj, O., Simović, R., Calculation of Scattered Radiation around a Patient Subjected to the X-Ray Diagnostic Examination, in Recent Advances in Multidisciplinary Applied Physics (Ed. A. Mendez-Vilas), Elsevier, 2005, pp. 751-756
[8] Marković, S., Simović, R., Ljubenov, V., Ilić, R. D., Spectral Albedo of Photons of Initial Energies below 100 keV, *Nuclear Technology & Radiation Protection, 22* (2007), 1, pp. 40-47
[9] Ljubenov, V., Simović, R., Marković, S., Ilić, R. D., Number Albedo of Low-Energy Photons for Water, Aluminum, and Iron, *Nuclear Technology & Radiation Protection, 22* (2007), 1, pp. 48-53
[10] Ljubenov, V., Simović, R., Marković, S., Ilić, R. D., Photon Scattering and Reflection in Medical Diagnostic Energy Domain, *Nuclear Technology & Radiation Protection, 23* (2008), 1, pp. 31-36
[11] ***, MCNP^{TM} – A General Monte Carlo N-Particle Transport Code, Version 4C, LA-13709-M, Manual, (Ed. J. E. Briesmeister), LANL, 2000
[12] Ilić, R. D., FOTELP-2K3, Photon, Electron, and Positron Monte Carlo Transport Simulation, IAEA 1388, OECD NEA Data Bank, 2002
[13] ***, PENELOPE-2005, A Code System for Monte-Carlo Simulation of Electron and Photon Transport, NEA-1525, 2005
[14] Marković, S., Backscattered Radiation Field in Diagnostic Radiology Contrast Techniques, Ph. D. Thesis, Faculty of Electrical Engineering, University of Belgrade, Belgrade, 2008
[15] Mashkovich, V. P., Ionizing Radiation Shielding – Manual (in Russian), Energoatomizdat, Moscow, 1982
[16] Marković, S., Ljubenov, V., Simović, R., A Semiempirical Formula for the Angular Differential Number Albedo of Low-Energy Photons, *Nuclear Technology & Radiation Protection, 20* (2005), 2, pp. 28-34

Received on June 4, 2010
Accepted on September 6, 2010

**Владан ЉУБЕНОВ, Родољуб Д. СИМОВИЋ,
Српко МАРКОВИЋ, Радован Д. ИЛИЋ**

УКУПНИ КОЕФИЦИЈЕНТИ РЕФЛЕКСИЈЕ НИСКОЕНЕРГЕТСКИХ ФОТОНА ПРЕДСТАВЉЕНИ КАО УНИВЕРЗАЛНЕ ФУНКЦИЈЕ

У раду је испитана могућност приказивања укупних честичних и енергетских коефицијената рефлексије нискоенергетских фотона заштитних материјала у облику универзалних функција. Анализа се заснива на резултатима Монте Карло симулација фотонске рефлексије који су добијени програмима MCNP, FOTELP и PENELOPE. Симулиран је вертикални продор уског моноенергетског снопа фотона јединичног интензитета са почетним енергијама од 20 keV до 100 keV, у равне хомогене мете начињене од воде, алуминијума и гвожђа и израчуната је честична и енергетска рефлексија. Приказани су албедо коефицијенти у зависности од почетне енергије фотона, од вероватноће расејања фотона на велики угао и од средњег броја расејања фотона. Показано је да једино албедо коефицијенти дати у зависности од средњег броја расејања фотона имају облике универзалних функција, које су одређене применом методе најмањих квадрата.

Кључне речи: рефлексија фотона, укупни бројни албедо, укупни енергетски албедо, Монте Карло метода, средњи број расејања фотона