A STUDY OF HIGH-FREQUENCY PROPERTIES OF PLASMA AND THE INFLUENCE OF ELECTROMAGNETIC RADIATION FROM IR TO XUV

by

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On the basis of calculated values for the conductivity in an external electric field, we determined the high-frequency characteristics of plasmas under extreme conditions (e.g. dense plasma focus device). The examined range of frequencies covers the IR, visible, UV, XUV up to X regions and the considered electronic number density and temperature are in the ranges of $10^{21} \text{cm}^{-3} \leq N_e \leq 5 \times 10^{24} \text{cm}^{-3}$ and $2 \times 10^4 \text{K} \leq T \leq 10^6 \text{K}$, respectively. The data obtained using this method are important for plasma focus research, laboratory plasma research, investigation of atmosphere plasmas of astrophysical objects like white dwarfs with different astrophysical compositions.

Key word: plasma focus, plasma, dynamic property, electromagnetic radiation

INTRODUCTION

The properties of dense plasmas under extreme conditions are of great interest in various research fields like nuclear physics, astrophysics, terrestrial physics, plasma physics [1-10]. Exploring and improving the new calculation possibilities, simulation techniques and the extension of numerous models in connection with the dynamic properties in the physics of high energy are in the focus of investigators nowadays [11]. Further investigation of strongly correlated plasma and investigations of its electronic properties remain an ongoing problem. For an example theoretical calculations and measurements of reflectivity are important because of its possible use as diagnostic tool in the area of high-density and high-energy physics [9, 12].

Determining high-frequency (HF) plasma properties in the homogeneous electrical field has been a subject of substantial investigation recently and is experiencing a real boom these days. The reason for this is the rapid development of experimental facilities in the field of extreme conditions aimed for demonstrating nuclear fusion in the laboratory [13, 14], for developing intense radiation sources with special properties [11, 15], and to investigate important and fundamental physics processes in astrophysics [16]. Unfortunately, the theoretical work and the explanations of these experimental measurements lag behind experimental research and do not follow it. Therefore, there was a need for more serious and even harder work in the field of theoretical work. There are two basic approaches to those theoretic studies: the generalized Drude-Lorentz model, [17], or the review [18], and the method of moments [19]. In addition, we have been working on the direct extension of the modified random-phase approximation for the calculation of the HF properties [20]. It should also be noted here, that we have to take advantage of growing high performance computing resources in the area of simulations and catch up and reach experimental research.

The aim of this work is determination of HF characteristics of non-ideal plasmas. All the presented values, like the electrical permeability and the coefficients of refractivity and reflectivity, as well as electromagnetic (EM) field penetration depth, are obtained by the help of the numerically calculated values for the dense plasma dynamic conductivity in an external HF electric field. The determination of the dynamic conductivity of a highly ionized plasma in a HF external electric field, as well as obtaining the other plasma pa-
rameters relies on the method presented in [20]. The method itself is based on the self-consistent field theory developed for the static plasma transport coefficients calculation, such as static plasma electric conductivity, etc. After the experimentally proven validity of the presented theory in the investigated domain of the electron densities and temperatures, the method was tested in the domain of more dense, non-ideal plasma i.e. was applied for calculation of conductivity of Degenerate B-type (DB) white dwarf atmospheres [21, 22] in wide area of electron densities and temperatures.

This work focuses on the determination of dynamic characteristics of dense plasmas as well as on introducing usable form of the presentation of calculated data (fitting formula) important for laboratory applications, e.g. dense plasma focus device (DPF), as well as for astrophysics modeling. All the data presented here will very soon be accessible through http://servo.aob.rs as web service and database.

**USED MODEL METHOD**

This paper considers a highly ionized plasma in a homogenous and monochromatic external electric field \( \mathbf{E}(t) = E_0 \exp(-i\omega t) \). According to [21] and [23], the dynamic electric conductivity of a strongly coupled plasma

\[
\sigma(\omega) = \frac{4e^2\omega m_e}{3m_0} \int_0^\infty \left[ \frac{d\omega(E)}{dE} \right] \rho(E) E dE = \sigma^{\text{res}}(\omega) + i\sigma^{\text{im}}(\omega)
\]

is presented by the expressions

\[
\sigma^{\text{res}}(\omega) = \frac{4e^2\omega m_e}{3m_0} \int_0^\infty \left[ \frac{d\omega(E)}{dE} \right] \rho(E) E dE
\]

\[
\sigma^{\text{im}}(\omega) = \frac{4e^2\omega m_e}{3m_0} \int_0^\infty \left[ \frac{\omega^2(E)}{1 + \omega \tau(E)} \right] \left[ \frac{d\omega(E)}{dE} \right] \rho(E) E dE
\]

Here \( \omega, m, E \) and \( \rho \) are the charge, mass, and energy of the free electron, \( \rho(E) \) is the one-electron states density in the energy space, \( \tau = \{\exp[\beta E - \beta \mu(1 + \lambda)]\}^{-1} \) the Fermi-Dirac distribution function, \( \mu \) – the chemical potential of the ideal gas of the free electrons with the density \( n_i \) and the temperature \( T, \beta = (k_B T)^{-1} \), and the relaxation time \( \tau(E) = \tau(E)/(1 - i \omega \tau(E)) \). In upper expressions \( \tau(E) \) is the “static” relaxation time and the method of determination of \( \tau(E) \) is described in the previous papers [21, 23, 24] in detail.

Other HF plasma characteristics can be expressed in terms of the quantities \( \sigma^{\text{res}}(\omega) \) and \( \sigma^{\text{im}}(\omega) \). In this way the plasma dielectric permittivity is shown as

\[
\varepsilon(\omega) = 1 + \frac{4\pi}{\omega} \sigma(\omega) = \varepsilon^{\text{res}}(\omega) + i\varepsilon^{\text{im}}(\omega)
\]

where \( \varepsilon^{\text{res}}(\omega) = 1 - (4\pi/\omega)\sigma^{\text{im}}(\omega) \) and \( \varepsilon^{\text{im}}(\omega) = (4\pi/\omega)\sigma^{\text{res}}(\omega) \). The coefficients of refraction \( n(\omega) \), and reflection \( R(\omega) \), are determined as

\[
n(\omega) = \sqrt{\varepsilon(\omega)} = n^{\text{res}}(\omega) + i n^{\text{im}}(\omega),
\]

\[
R(\omega) = \frac{n(\omega) - 1}{n(\omega) + 1}
\]

where, bearing in mind that

\[
|\varepsilon(\omega)| = \left[ (\varepsilon^{\text{res}}(\omega) + \varepsilon^{\text{im}}(\omega))^2 \right]^{1/2}
\]

the real and imaginary part of refractivity are given by

\[
n^{\text{res}}(\omega) = [0.5(\varepsilon^{\text{res}}(\omega) + \varepsilon^{\text{im}}(\omega))]^{1/2}
\]

and

\[
n^{\text{im}}(\omega) = [0.5(\varepsilon^{\text{res}}(\omega) - \varepsilon^{\text{im}}(\omega))]^{1/2}
\]

From here the equation for the reflectivity could be expressed as

\[
R(\omega) = \left[ \frac{1 + \varepsilon^{\text{res}}(\omega) - \sqrt{2} \sqrt{\varepsilon^{\text{res}}(\omega) + \varepsilon^{\text{im}}(\omega)}}{1 + \varepsilon^{\text{res}}(\omega) + \sqrt{2} \sqrt{\varepsilon^{\text{res}}(\omega) + \varepsilon^{\text{im}}(\omega)}} \right]^{1/2}
\]

In such a way, we determined the other parameter of interest i.e. the penetration depth of electromagnetic radiation into plasma, \( \Delta(\omega) \). This quantity is the skin-layer width determined as the inverse imaginary part of the electromagnetic field wave number \( \Delta(\omega) = (\varepsilon(\omega))^{-1} \) where \( \varepsilon(\omega) \) is the speed of light and \( n^{\text{im}}(\omega) \) is defined above.

Here, it is common to use a dimensionless coupling parameter, the plasma non-ideality coefficient \( \Gamma \), that characterizes the physical properties of the plasma. It is of especial importance in describing of dense, non-ideal plasmas, as ones considered in this paper. The mentioned parameter \( \Gamma = e^2/(4\pi n e^2) \) such characterizes the potential energy of interaction at average distance between particles \( a = (3/4\pi n e^2)^{1/3} \) in comparison with the thermal energy. The well-known Brueckner density parameter \( r_b = a/a_B \) is the ratio of Wigner-Seitz and Bohr radius.

**RESULTS AND DISCUSSION**

As the continuation of previous investigation, that is of interest for DB white dwarf atmospheres [21, 22] and high energy research, the calculations of the HF characteristics of plasma in wide area of electron densities and temperatures were conducted. The investigated areas are of interest for both of the laboratory experiments, e.g. DPF, and atmospheres of different stellar types like degenerate a-type (DA), or degenerate c-type (DC) white dwarfs.

The electrical permeability and the coefficients of refractivity and reflectivity of dense non-ideal plasmas are determined on the basis of numerically calculated values for the dense plasma dynamic conductivity in an external HF electric field. Here are considered electronic number density and temperature in the
ranges of $10^{21} \, \text{cm}^{-3} \leq N_e \leq 10^{24} \, \text{cm}^{-3}$ and $2 \cdot 10^8 \, \text{K} \leq T \leq 10^9 \, \text{K}$, respectively. The examined range of frequencies $\omega$ covers the region $0 \leq \omega \leq \omega_p$, where $\omega_p = (4\pi N_e e^2/m)^{1/2}$ is the plasma frequency.

Within the considered ranges of variation of electron density and temperature, the behavior of the HF plasma quantities with the increase of the $N_e$ and $T$ is as it is expected. This means that the behavior of those characteristics, with the increasing of $N_e$ and $T$, remains the same within the whole range of electron number densities and temperatures $10^{21} \, \text{cm}^{-3} \leq N_e \leq 10^{24} \, \text{cm}^{-3}$. The same conclusion is also related to all yielded parameters, including the penetration depth $\Delta(\omega)$. From the results it could be observed that for the long-wavelength radiation, i.e., frequency tends to zero, there is interesting dependence on temperature and frequency of skin-effect $\Delta$ at surface plot in fig. 1. Also, the reflectivity coefficient tends to 1, mirroring the plasma layer acts as a mirror.

Particularly, here we give special attention to analysis of reflectivity data because this quantity is very important in high energy density plasmas (measurements as an important diagnostic tool provide information about the density profile). In fig. 3 we compare the behavior of reflectivity coefficient $R$, determined in this paper, with the behavior of the corresponding quantities calculated in [25] for $r = 5$ and $\Gamma = 0.1$, $\Gamma = 0.5$, $\Gamma = 1$ and also for $\Gamma = 0.2$ and $r = 0.4$, $r = 1$ and $r = 2$. Here $r$ and $\Gamma$ are previous defined Brueckner density and non-ideality parameters, respectively. The corresponding curves showing theoretical data from [25] are presented by full, dashed, and dotted lines. Figure 3 shows a qualitative agreement (the differences are less or close to 10-25 percent) of our results with the results of [25] in the whole region of $\omega$ and for all values of non-ideality parameters. By analyzing the behavior of corresponding curves in figs. 2 and 3 one can see the tendency that increase in the density and coupling parameters leads to a decrease in the reflection index. Then, in fig. 3 it is seen that at frequencies $\omega < \omega_p$ the reflection occurs most effectively and at $\omega \to \omega_p$ the plasma becomes much more transparent.

In fig. 4 is presented a surface plot of the plasma reflectivity for different values of density as a function of temperature and frequency. With this figure it is possible to map the region of high non-ideality. For a given density the appropriate panel from fig. 4 can be selected and the value of $R$ found for the chosen pair of temperature and frequency.

In order to more simplify the presented results for the further use a fitting procedure is exploited to get the plasma reflectivity from the temperature and frequency for various values of electron density. For all analyzed densities the expression is given by

$$R(x, y) = a + bx + cy + dx^2 + ey^2 + fx y + gx^3 + hy^3 + ix y^2 + jx y^3$$

(7)

where $x = \omega/\omega_p$, ratio of frequency and plasma frequency ($0 \leq \omega \leq \omega_p$), $y = T$, temperature and $a-j$ are calculated parameters. These parameters were computed.

![Figure 1](image1.png)

Figure 1. Left: surface plot of the skin layer depth for $N_e = 10^{21} \, \text{cm}^{-3}$ and $2 \cdot 10^8 \, \text{K} \leq T \leq 10^9 \, \text{K}$; right: same as in left but for $N_e = 10^{24} \, \text{cm}^{-3}$

![Figure 2](image2.png)

Figure 2. Down: the plasma reflectivity for $N_e = 10^{21} \, \text{cm}^{-3}$ and $2 \cdot 10^8 \, \text{K} \leq T \leq 10^9 \, \text{K}$; up: same as in lower panel but for $N_e = 10^{24} \, \text{cm}^{-3}$

![Figure 3](image3.png)

Figure 3. The plasma reflectivity as a function of $\omega/\omega_p$ for various values of quantities $r$ and $\Gamma$: present results are shown together with results from [25]
from a standard 3-D fitting procedure, (best-fit value i.e., r-Square close to one) and their values as the function of density are shown in tab. 1. We must emphasize that those expressions give good results for values of frequencies in the region $0 < \omega < 0.3 \omega_p$ although they have to be applied with caution beyond this region.

**Some HF quantities:** The behavior of some HF quantities for various plasma conditions is illustrated here although it is not the main topic of our research. The reason of its representation is that it enables determination of other transport properties. The electrical permeability and the coefficients of refractivity of dense non-ideal plasmas are determined on the basis of numerically calculated values for the dense plasma dynamic conductivity in an external HF electric field. Here we will present the most extreme cases which we think are the most interesting for readers (all other data covering the ranges of $10^{21}$ cm$^{-3} \leq N_e \leq 10^{24}$ cm$^{-3}$ and $10^4$ K $< T < 10^6$ K will be presented in database).

On fig. 5 the real and imaginary part of refraction coefficient for density $N_e = 10^{24}$ cm$^{-3}$ in temperature region $2 \times 10^4$ K $\leq T \leq 10^6$ K is presented as a function of frequencies $\omega$. From this figure one can see that the behavior of this HF quantity is similar to the investigated ones in upper material.

Further analysing the presented results it could be observed that for the long-wavelength radiation, i.e., frequency tends to zero, even for most extreme cases the imaginary part of the dielectric function diverges, i.e., $\varepsilon(\omega \rightarrow 0)$ is much greater than real part of the dielectric function (fig. 6).

**Astrophysical relevance:** As we noted in the above text the investigations of HF characteristics is important for research of atmosphere plasmas of astrophysical objects like white dwarfs with different atmospheric compositions (DA, DC etc.), and some other stars (M-type red dwarfs, Sun, etc.). In order to demonstrate this astrophysical relevance fig.7 is presented.

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**Table 1. Parameters (from a to j) needed for $R(x,y)$ fit given by eq. 7, as a function of electron density where $x=\omega\omega_p$ and $y=T$**

<table>
<thead>
<tr>
<th>$F_r/N_e$</th>
<th>$10^1$ [cm$^{-3}$]</th>
<th>$10^2$ [cm$^{-3}$]</th>
<th>$10^3$ [cm$^{-3}$]</th>
<th>$10^4$ [cm$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.935052097</td>
<td>0.947234943</td>
<td>0.967203184</td>
<td>0.981546732</td>
</tr>
<tr>
<td>$b$</td>
<td>-1.317415548</td>
<td>-1.44143753</td>
<td>-1.25315644</td>
<td>-0.863649267</td>
</tr>
<tr>
<td>$c$</td>
<td>7.29876E-07</td>
<td>4.12914E-07</td>
<td>9.29373E-08</td>
<td>-3.83153E-10</td>
</tr>
<tr>
<td>$d$</td>
<td>2.200777151</td>
<td>2.59112663</td>
<td>2.623576736</td>
<td>2.2174305</td>
</tr>
<tr>
<td>$e$</td>
<td>-1.81598E-12</td>
<td>-9.04495E-13</td>
<td>-1.02346E-13</td>
<td>2.41917E-14</td>
</tr>
<tr>
<td>$f$</td>
<td>1.97343E-06</td>
<td>1.52641E-06</td>
<td>6.20522E-07</td>
<td>1.38124E-08</td>
</tr>
<tr>
<td>$g$</td>
<td>-1.535152818</td>
<td>-1.760714503</td>
<td>-1.851589254</td>
<td>-1.69034055</td>
</tr>
<tr>
<td>$h$</td>
<td>1.15411E-18</td>
<td>5.43242E-19</td>
<td>3.83954E-20</td>
<td>-1.3866E-20</td>
</tr>
<tr>
<td>$i$</td>
<td>-1.17436E-12</td>
<td>-6.97197E-13</td>
<td>-1.04538E-13</td>
<td>6.15697E-14</td>
</tr>
<tr>
<td>$j$</td>
<td>-3.59279E-07</td>
<td>-4.84845E-07</td>
<td>-3.40011E-07</td>
<td>-6.11506E-08</td>
</tr>
</tbody>
</table>
here. In fig. 7(a) is illustrated, by surface plot, the conductivity as a function of the logarithm of Rosseland opacity \( \log \tau \) for DB white dwarf atmosphere models with logarithm of surface gravity \( \log g = 8 \) and various effective temperatures from \( T_{\text{eff}} = 1.2 \times 10^4 \) K to \( T_{\text{eff}} = 3 \times 10^4 \) K. Figure 7(a) shows that increasing the white dwarfs effective temperature, at a fixed value of the logarithm of Rosseland opacity, leads, as a consequence.
quence, to an increase of conductivity. Also, it is illustrated here in fig. 7(b) plot of HF conductivity for log $\tau = 2$ for DB white dwarf atmosphere models $T_{\text{eff}} = 3 \times 10^4$ K and log $g = 8$ which corresponds to electron density: $\sim 10^{19}$ cm$^{-3}$ and temperature: $\sim 10^4$ K. From the presented fig. 7(a,b), it can be observed a regular behaviour of the conductivity considering the characteristics of DB white dwarf atmospheres.

CONCLUSIONS AND PERSPECTIVES

In this paper we calculated the dynamic characteristics of plasmas of increased non-ideality under the influence of IR to XUV, electromagnetic radiation. The presented data covers wide region of electron densities and temperatures. These results, especially fit formula, can be applied in the experiments of DPF, high pressure discharge, shock waves etc., where strongly non-ideal plasmas, including extremely dense plasmas, are created. The developed method presents a useful tool for the research of white dwarfs with different atmospheric compositions (DA, DC etc.), as well as for some other stars (M-type red dwarfs, Sun etc.). In the near future we aim to further improve analysis using GEANT code [26]. Also, our plan is to present the results obtained during this investigation in database which can be accessed directly through http://servo.aob.rs as a web service similarly to the existing MoID and E-MOL databases http://servo.aob.rs/mold, http://servo.aob.rs/emol/ [27, 28].

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AUTHORS’ CONTRIBUTIONS

The calculations and simulations were carried out by V. A. Srecković, who wrote the manuscript. M. S. Dimitrijević, D. M. Šulić, and Lj. M. Ignjatović participated in editing and revising of the manuscript. All authors discussed the results and commented on the manuscript.

REFERENCES

V. A. Srećković, et al.: A Study of High-Frequency Properties of Plasma and the...


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ИСПИТИВАЊЕ ВИСокоФРЕКВЕНЦИЈСКИХ ОСОБИНА ПЛАЗМЕ И УТИЦАЈ ЕЛЕКТРОМАГНЕТНОГ ЗРАЧЕЊА У ДОМЕНУ ОД IR ДО XUV

На основу израчунатих вредности проводности у спољашњем електричном пољу, одређене су високофrekвенцијске карактеристике плазме у екстремним условима (на пример, код плазма фокусних уређаја). Испитивани опseg фrekвенција покрива IR, видљиву, UV, XUV све до X редина и разматране су електронске густине и температуре у распону 10^{21} \text{ cm}^{-3} \leq N_e \leq 5 \cdot 10^{24} \text{ cm}^{-3} и 2 \cdot 10^4 K \leq T \leq 10^6 K, респективно. Подаци добијени коришћењем ове методе важни су за истраживања плазма фокусних уређаја, за истраживања лабораторијске плазме, а такође и за истраживања плазме астрофизичких објеката, као што су бели патуљци са различитим атмосферским композицијама.

Кључне речи: плазма фокус, плазма, динамичко својство, електромагнетно зрачење