

Interpretation of a Discovery

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Abstract: The paper presents the development of the theory of asynchronous motors, since Tesla's discovery until the present day. The theory of steady state, as we know it today, was completed already during the first dozen of years. That was followed by a period of stagnation during a number of decades, when the theory of asynchronous motors was developed only in the framework of the general theory of electric machines, which was stimulated by the problems of the development of synchronous generators and big electric networks. It is only in our time that this simple motor, which was used for a long time just to perform crude tasks, became again the inspiration for the researchers and engineers who enabled it, with the help of power electronics and semi-conductor technology, to be used in the finest drives.

Keywords: Asynchronous motor, theory, historical survey.

1 Introduction

The asynchronous motor was discovered by the end of the next to last decade of the 19th century, as an important complement to a broader idea – the idea of polyphase system for the production, transmission, and use of electric power. At that time, the system with direct current was dominant for those purposes, but it was quite clear that the losses in transmission represented the main limitation of its expansion. The idea to use, instead of commutators by dynamos of direct current, sliding rings and to get in that way alternate current whose voltage could be increased by such a simple gadget as is the transformer, for transmission, and then again decreased for use, was adopted in the branch of economy which was, at that time, the main consumer of electric power – electric lighting. But that did not solve the question of electric transmission of mechanic power, in other words, there were no good motors for alternate current.

The inventor of the system with alternate current, which solved at the same time the question of motors, Nikola Tesla, explained, in May, 1888 in the Institute of American Electrical Engineers, the substance of the idea in the best possible way [1,2].

“In our dynamo machines, as it is well known, we produce alternate currents, which we rectify by commutators, a complicated device for which we

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may rightly say that it represents the major source of troubles which appear in the function of such machines. However, such rectified currents can not be used in motors, but must – again with a similar unreliable device – be transformed into their original state of alternate current. The function of the commutator is quite external and does not influence, in any way, the internal function of the machines. Therefore, *all machines are, in fact, machines with alternate current* and the currents appear as direct current machines only in the external current circuit during their passage from the dynamo into the motor. Quite simply, bearing that fact in mind, the alternate currents by themselves would be recommended as the direct application of electric power while the application of direct currents would be justified only if we would have dynamo machines which would primarily produce such currents and motors which would be directly driven by such currents.

“However, the function of the commutator in the motor is two-fold: first, it produces the changes of the direction of the current in the motor, and, second, it achieves automatically the progressive movement of the poles of one of its magnetic elements. Let us, therefore, assume, that both those unnecessary functions in the systems, i.e. the rectifying of the alternate current in the dynamo and the creation of the change of the direction of the direct current in the motor are eliminated, then, in order to produce the rotation of the motor it would be only necessary to achieve the *progressive movement of the poles* of one of its elements and the question is automatically imposed how that operation could be achieved by direct action of alternate currents.”

Then, Tesla explained how he achieved that “progressive movement of the poles” by making a rotating field with immovable elements and without contacts, by using polyphase currents and how he created the desired commutatorless system for the production, transmission and use of electric power, including drives. In fact, that was the presentation of the contents of a series of his patents which had been registered during the previous 1887 year. He presented, inter alia, the bases of the construction and of the behavior of the motors which we now call asynchronous, and he interpreted verbally, with the help of experiments and pictures, their function based on the principle of induction. The motors which did not have to have, at their moving parts, commutators or sliding rings, nor any connection with external circuits, and which behaved, in their function, as the then dominant direct current motors with insular excitation.

That time was predestined for the invention of the asynchronous motor. The rotation of the astronomer Arago from 1824 was known, then there were researches in the field of magnetic-electric induction and the explanation of that phenomenon at the basis of induced eddy currents of Michael Faraday in 1831, and the observations and experiments of a number of other physicists. However,

it seems that until 1879, nobody imagined that Arago's rotation could be used in the construction of motors [3] In that year, Baily induced Arago's rotation not by a moving magnet, but achieved it "by progressive movement of poles" with immovable electric circuits, but with the help of switches which were successively put on and off. Somewhat later, the Frenchman Depré achieved some improvements in the same direction, but these were only steps towards the future induction motor. Only in 1888, i.e. after the registration of Tesla's patents (a little before the mentioned Tesla's lecture), Ferraris presented to the public [4] in Turin his way of production of rotating field and rotation induced by induction by means of *immovable and contactless* apparatus – by means of two-phase alternate currents. That was not a power system which would include the dynamo, the transmission and the motor, like Tesla's system, which was earmarked for replacing the entire existing electric power system with direct current, but it was an induction motor. On the contrary, Ferraris himself concluded in the same lecture that "the apparatus based on that principle could not be of any commercial importance as motor."

It seldom happened in technology, maybe only in the recent times, in the era of the discovery of semi-conductors, that an invention was adopted so quickly by engineers and businessmen, both on the New and on the Old continents, and introduced rapidly in practice. Soon afterwards, many people started working eagerly on the improvement of the new motor. Renowned magazines published hundreds of articles with reports about the construction of the new system, about the construction of new, improved motors, about the world exhibitions in which big industrial firms competed with theoretical lectures about the explanation of the behavior of the new motor and about the ways of its calculation. Within ten years only, the decision was taken and all basic technical principles necessary for the practice of that time were solved.

2 First Theory

Tesla came to his discoveries by thought and experiments, using his extraordinary intuition and his capability to identify two things simultaneously: what the humanity needs, and what the secrets of the nature, discovered and undiscovered at that time, may offer to humanity. He presented to the public his patents and the mentioned lecture only when, after many years of stubborn experimenting, he was convinced that the system was practically usable. He gave the explanation of the function of his asynchronous motor only verbally, in one of his first patents [5] (and later, somewhat in more detail in the following way:

"If such motors are not loaded, and rotate freely, the rotation of the armature is approximately synchronous with the rotation of the poles of the field and under such conditions there appears in the coils a very weak current, but if load

is added the velocity will tend to decrease and the current in the coils will become stronger and the torque will increase proportionally.”

However, with the exception of two trigonometric formulas by which he wanted to explain better, in his lecture, the principle of the creation of the rotating field by two-phase currents, he gave no theoretical analyses, neither for the explanation of the behavior of the motor, nor for the purpose of its calculation.

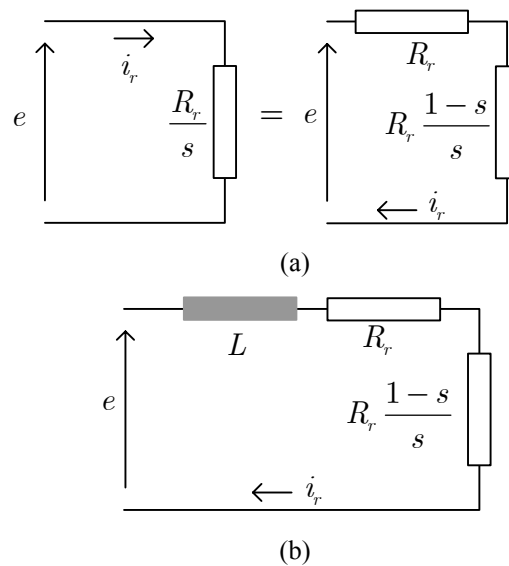


Fig. 1 – Equivalent scheme of the rotor of the asynchronous motor: a) According to Ferraris’s analysis (1888); b) With the addition of inductance (1891).

The first analysis was given by Ferraris in the mentioned lecture and it may be considered as the beginning of the creation of the theory of asynchronous motors. Although with some shortcomings, it gave the first bases for the further development of the theory. Starting from the fact that the electromotive force in the rotor is constant, i.e. neglecting the drops of voltage in the stator (which really does not enter into the substance of the function of asynchronous motors), and assuming that the induced current is proportional to the sliding, which is equivalent to neglecting the inductive drop of voltage in the rotor (which may be accepted only by small sliding, when the frequencies in the rotor are small), Ferraris obtained the formula for the mechanical power and the torque of the motor. There is, in it, just a part of the presently well known equivalent scheme of asynchronous motors, true, a substantial part, but a part which conceals many other equally important phenomena (Fig. 1a).

Out of this scheme, where e is the induced electromotive force in the rotor, and R_r the resistance of the rotor, it comes out that the rotor current (with presently used marks):

$$i_r = \frac{e}{R_r / s}, \quad (1)$$

i.e. it is proportional to the relative sliding s , the losses (in the copper of the rotor):

$$p_{Cur} = \frac{e^2}{R_r} s^2, \quad (2)$$

the mechanical power (after deducing the losses)

$$p = R_r \frac{1-s}{s} i_r^2 = \frac{e^2}{R_r} (1-s)s \quad (3)$$

and the torque:

$$m = \frac{p}{\omega} = \frac{e^2}{R_r \omega_s} s, \quad (4)$$

(where ω is the angular velocity of the rotor, and ω_2 that of the rotating field), so we get that the torque is simply proportional to the sliding, in synchronism zero (which is correct), and highest at start (which is erroneous, except for a determined R_r). Besides, as the resistance of the rotor is smaller, the starting torque is bigger (incorrect). The curve of the torque in the function of the velocity has no extremes, but the curve of power has an extreme and that extreme is situated at the half of the velocity.

Out of this, Ferraris concludes: "When the mechanical power is maximum, or when $\omega = \omega_2 / 2$, we have $p = p_{cur}$ so that the mechanical work is equal to the heat developed." And further on: "These calculations and the experimental results confirm a priori the obvious conclusion that the apparatus based on that principle can not be of commercial importance as a motor and it would be useless to study that problem..." It is a paradox that Ferraris really discovered the asynchronous motor and even posed the foundations of its theory, to conclude, at the basis of the theory, that the motor he invented is useless! Where is the main Ferraris's mistake?

If you add the inductive drop of voltage due to some still undetermined self-induction L (only much later that inductance was defined as the inductance of leakage), which Ferraris did not take into account, things become qualitatively different. So, one gets

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$$i_r = \frac{e}{\sqrt{(R_r/s)^2 + (\omega_s L)^2}}, \quad (5)$$

$$p = \frac{e^2}{R_r(1 + s\omega_s L/R_r)}(1-s)s, \quad (6)$$

$$m = \left(\frac{e}{\omega_s}\right)^2 \frac{\omega_r R_r}{R_r^2 + (\omega_r L)^2} = \frac{e^2}{R_r \omega_s} \frac{s}{1 + (s\omega_s L/R_r)^2}. \quad (7)$$

Out of these formulas, it follows that the torque *has a maximum* by sliding (in motor régime)

$$s = s_p = \frac{R_r}{\omega_s L} \quad (8)$$

and the power by

$$s = s'_p = s_p \left(\sqrt{s_p^2 + 1} - s_p \right). \quad (9)$$

The maximum power occurs at a speed which is a little bigger than the speed at maximum torque – i.e. in stable region. For, for instance $s_p = 0.2$, $s'_p = 0.164$ and that maximum power represents only 0.328 of the power which is obtained as maximum power according to Ferraris's formula.

However, the question arises how the existence of leakage becomes the key factor for the use of the principle of induction for the construction of a practically usable motor, when it is well known that the influence of leakage is only negative (it decreases the torque, especially the starting torque)? The answer is that the erroneous evaluation of Ferraris is due to something quite different, and not in the neglecting of leakage. Namely, it can be easily understood that in both cases

$$\eta = \frac{p}{p + p_{Cur}} = 1 - s \quad (10)$$

and then the question arises why Ferraris insisted that it is considered only at maximum power? That is not the maximum power for the given material, but it is the velocity below which the power does not increase, but decreases with the decrease of velocity. Therefore, his basic mistake is not due to the fact that he neglected the inductive drops of voltage (which is, as it is said, equivalent to the assumption of proportionality of current and sliding), but to the erroneous criterion for the optimum function of the motor. It could be concluded, from his formula that with sliding which is small enough, a good usefulness could be obtained. True, the power would be substantially smaller than the maximum, but

it will correspond to what can be drawn out of the given material, bearing in mind the saturation of the magnetic core and the permitted heating of the machine.

Later on, there were many theoretical discussions about that question in the literature [6]. However, it is interesting to note that Dolivo-Dobrovolski, in his historical survey of 1916 [7] states that he got the first encouragement for early studies of asynchronous motors from Ferraris, not from Tesla, but that starting from his experience in the construction of motors with direct current, he immediately rejected his way of thinking which resulted in the well known pessimist conclusion.

Immediately after Tesla's and Ferraris's lectures a relatively big number of engineers and theoreticians all over the world worked on the explanation of the new invention. The engineers, mainly employed in big firms, as for instance AEG in Germany, Oerlikon in Switzerland, Westinghouse in the USA, worked on the construction and improvement of prototypes of asynchronous motors and discovered, mainly by experimental work, and tried to explain, more or less intuitively, the properties of the new motor with the main objective to obtain the rules for their calculations, which would replace the then dominant method of cut-and-try. The theoreticians did the same thing, of course, in their own way, using the past theoretical knowledge about electromagnetism. At the very beginning, two important questions arose general attention: the question of start and run of the motor and the question of the treatment of the inductance in the motor. The first question is quite a practical one, but the second, as it will be seen, was more academic than practical, and it arose many discussions about the approach to the elaboration of the theory of asynchronous motors.

3 The Starting Torque

The first theoretical analyses which surpassed Ferraris's analyses from 1888, appeared only in 1891. In that year, the French engineers Hutin and Leblanc [8] and the English professor Duncan [9] published their analyses in which they took into account the inductive drop of voltage in the rotor, and obtained formulas which gave, inter alia, the qualitatively correct characteristic torque/velocity. As a difference to Ferraris's analysis, which gave a characteristic similar to that of direct current motor with constant excitation., that formula determined a maximum at a given speed, not much smaller than the synchronous speed, below which the torque decreased. In that way, the earlier observed difficulties which appeared at the start of a motor under load, could be explained. Moreover, thanks to the improved formulas, one could explain the earlier observed paradox property of asynchronous motors that the increased resistance of the rotor contributes to the increase of the starting torque, with the decrease of the starting current. In formula (7) one can clearly see the changing

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influence of the reactance $\omega_r L$ on the torque, with the change of velocity. In the vicinity of synchronism, the sliding, i.e. the rotor frequency ω_r is small, so that the active drop of voltage due to the resistance of the rotor is dominant. The torque is approximately proportional to the sliding and increases with the decrease of rotor resistance. That is, anyway, the conclusion of Ferraris's analysis (according to equation (4)). However, at start, when the frequency of the rotor is big, it may happen that the reactive drop of voltage dims very much the active one, and we get the opposite situation: bigger rotor resistance - bigger torque.

Dolivo-Dobrowolsky, in his remembrances at the lecture in the Frankfurt Electrotechnical Society in 1916 [7], claims that he found that property of the motor already in 1898, when he saw that his second experimental motor, much bigger than the first one (around 5 HP) had big difficulties when starting. He soon concluded that one of the differences between those two motors lied in the fact that the second one had a much bigger cross-section of the conductors in the rotor and that, maybe, the short-circuited rotor was "too much short-circuited" ("zu sehr kurzgeschlossen"). By a researcher of experimental type, like Dolivo-Dobrowolsky (very much similar to Tesla) there is no long thinking, and the "maybe" is immediately checked: he took out the thick bars from the rotor and wound into the empty openings thin copper conductors with different numbers of windings. As end result, he got a motor with quite satisfactory starting torque. After that he concluded that such a motor is not functioning economically, when that is most important, i.e. at small sliding. In the next step, he made a motor with sliding rings and variable external rotor rheostat, so he solved both problems and, besides, concluded that even the speed of the motor may be adjusted.

Although he solved the practical problem, Dolivo-Dobrowolsky gave an erroneous, or at least quite vague explanation of that property of asynchronous motors. Namely, because of the small resistance of the rotor, the currents at start are large, therefore the currents in the stator as well. By such large currents the amperwindings of the stator are big, so the force lines find secondary passages, and hardly reach the rotor. "Therefore, at too much short short-circuits and consequently at too high rotor currents, the field is rejected aside, and therefore, the torque decreases because of the lack of field." The complete theoretical explanation came, as we saw, only two years later.

This episode represents a good example of the advantage of intuitive approach to research, in comparison with logic-analytic. The practical problem was solved, because the researcher felt where the solution could be found, and he found it by experiments, with a lot of tries and with hard work. Afterwards, it is not important whether the solution is correctly explained – it is there.

Theoreticians will explain that later with more precision, they will give the quantitative relations which are indispensable for optimum constructions, and without which it is impossible to predict the behavior before the often expensive construction.

Dolivo-Dobrowolsky, working in AEG in Berlin, for some time in cooperation with Brown from Oerlikon in Switzerland, contributed very much to the development of the construction of asynchronous motors. Although he appeared quite often in the literature, one could seldom find formulas in his articles, except in one case when he tried to prove, in a wrong way, that a more regular torque is obtained with the increase of the number of phases. He claimed that the “Tesla-motor” was not good because of too big pulsations of the torque in two-phase systems, and that is why he worked with three-phase systems [10]. As it is well known, the three-phase system was adopted, but not because of the pulsations.

4 Mechanical Characteristic

The mentioned work by Hutin and Leblanc presented a detailed theory of the motor of the asynchronous motor, (which the authors call “a machine of different kind”) which probably results in the first, under certain conditions, correct formula for the torque of the motor as a function of sliding. They studied the two-phase motor supplied from a single-phase source, with an auxiliary phase through a capacitor, but with the assumption that the two-phase currents have the same amplitudes and the ideal lagging of phase of one fourth of a period. Translated into the modern way of notation, the formula for the mechanical characteristic, drawn from their formula for mechanical power of the motor, reads:

$$m = 2P \frac{\omega_r R_r}{R_r^2 + (\omega_r L_r)^2} M^2 I_s^2 \quad (11)$$

(M is the mutual inductance, L_r the self-inductance (total) of the rotor, I_s the current in the stator (effective value), and P the number of pairs of poles) which is correct, but with little practical value, for it is understood that the stator current remains constant at different sliding. (This formula got its practical value only recently, when the supply with forced currents became possible thanks to the power electronics and automatic control). Moreover, the authors analyze further the function of the motor under the assumption that in a certain way, by adjusting the rotor resistance, the condition for maximum $R_r = \omega L_r$ is always kept. Still, their analysis gave the first theoretical explanation of the positive effect of the increased rotor resistance on the starting torque, and of the negative effect of the “self-induction of the moved part of the machine”.

The first more complete theoretical analyses which resulted in the formulas which are still found in the textbooks appeared in 1893, thanks to Arnold [11], Blondel [12] and Behn-Eschenburg [13]. This last author gave at the end of the deduction the complete formula for the mechanical characteristic of the motor. There he took implicitly into account (via the self-inductance of the stator L_s and of the rotor L_r , the interinductance M , and $\sigma = 1 - M^2 / (L_s L_r)$ is total leakage coefficient, both the magnetizing current and the leakage. With the neglecting of the stator resistance, which was also taken into account by Behn-Eschenburg, we show his formula for the torque (by phase and pair of poles).

$$m = \frac{M^2 U_s^2 \omega_r R_r}{(\omega_s L_s R_r)^2 + (\omega_s L_s \omega_r L_r \sigma)^2} = \left(\frac{U_s}{\omega_s} \right)^2 \left(\frac{M}{L_s} \right)^2 \frac{\omega_r R_r}{R_r^2 + (\omega_r \sigma L_r)}, \quad (12)$$

which is not different from the present one. Although at that time there was still no mention of the equivalent scheme, the entire deduction, and this formula are in accordance with the equivalent scheme on Fig. 2.

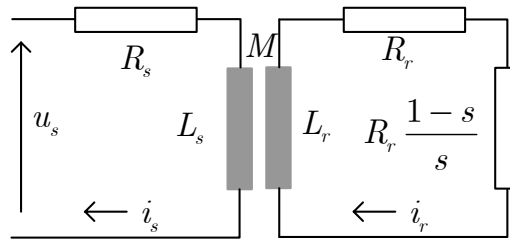


Fig. 2 – Equivalent scheme of asynchronous motors following the analyses of Arnold and Blondel (1893).

As it may be seen, differently from Hutin and Leblanc, it takes as constant the supply voltage U_s and not the stator current L_s . In quantitative respect, there exist big differences between the two formulas. First of all, the inductance σL_r is responsible for the inductive drop of voltage in the rotor, instead of L_r which is smaller for an entire order of magnitude, for there it is the question of the inductance of leakage, and not of the total self-inductance of the rotor. The maximum torque appears at substantially higher slidings, while the rotor resistance, which is necessary for the maximum starting torque, is much smaller.

5 leakage and Common Flux

We already saw that the theoreticians understood, already at the very beginning of the eight-year “period of analysis of asynchronous motors” (according to Lamm) from 1891 to 1897, the importance of the leakage indu-

stance for the behavior of the asynchronous motor. After all, that was the main point in the revision of the Ferraris theory.

However, the definition of the leakage inductances in its present meaning, and of the notions linked to them, like the common and leakage flux and the magnetizing current appeared only later, after the debates between two groups of theoreticians, which continued partially in the next century, as well. Until now, the mentioned theoreticians (Duncan, Hutin, Leblanc, Arnold Blondel, Behn-Eschenburg) started from Maxwell's equations from 1865 [14], which concerned the magnetically coupled electric circuits and in which self-inductances and interinductances appeared as coefficients of proportionality between the total fluxes and various currents. There was no mention of leakage inductances, and their role was taken by the coupling coefficients and possibly by the coefficients of total leakage.

Contrary to that, another group of theoreticians, lead by Steinmetz, Kapp, Hopkinson and others, started from the already elaborated (at that time) theory of transformers and certain experiences in the construction of direct current ("dynamo") machines, which constructed the picture of the phenomena in those apparatuses on the basis of the common flux which is excited by the magnetising current and of the leakage fluxes excited by the current in every winding separately and which are not included in other windings. In the mathematical sense, the difference between these two approaches may look, at first glance, quite formal, because out of the equations of the fluxes, which characterize (in the case of two coupled windings with indices 1 and 2) the Maxwellian approach:

$$\begin{aligned}\psi_1 &= L_1 i_1 + M i_2 \\ \psi_2 &= M i_1 + L_2 i_2\end{aligned}\quad (13)$$

with primary and secondary self-inductance L_1 and L_2 and interinductance M , it is easy to pass to the other approach by simple rearrangement:

$$\begin{aligned}\psi_1 &= M(i_1 + i_2) + (L_1 - M)i_1 \\ \psi_2 &= M(i_1 + i_2) + (L_2 - M)i_2\end{aligned}\quad (14)$$

and by the introduction of new definitions of parameters and values. Namely, when one defines the notion of the primary and secondary leakage inductance according to formulas:

$$\begin{aligned}\Lambda_1 &= L_1 - M \\ \Lambda_2 &= L_2 - M\end{aligned}\quad (15)$$

(for L_1 is always $> M$ and $L_2 > M$) according to the formulas:

$$\begin{aligned}\lambda_1 &= \Lambda_1 i_1 \\ \lambda_2 &= \Lambda_2 i_2\end{aligned}\quad (16)$$

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then the magnetizing current and the common flux according to formulas:

$$\begin{aligned} i_m &= i_1 + i_2 \\ \Psi_m &= M i_m \end{aligned} \quad (17)$$

we get the equations which express the new picture:

$$\begin{aligned} \Psi_1 &= \Psi_m + \lambda_1 \\ \Psi_2 &= \Psi_m + \lambda_2 \end{aligned} \quad (18)$$

The united currents of the primary and secondary (or, more correctly, the united amperewindings) create a common flux, a flux which includes both coils. Besides, each of the coils creates its own flux by its current (leakage flux) which is not included in the other coil.

Such an approach has a number of advantages: (1) we get a clearer physical representation with the idea that the fluxes defined in that way may be represented physically (now they are called fictive), (2) in the middle with iron, and especially in power transformers and electrical machines we talk about, there is a big quantitative difference between the magnetic resistance met by the common flux (mainly through iron with two air gaps) and the resistance met by the leakage fluxes (mainly through air) so, by knowing those passages, the geometrical relations and the conductivity of the magnetic medium, the resistances may be better determined quantitatively, (3) the losses in iron and due to eddy currents, and the influence of the variable capacity of iron (magnetic saturation) are linked only to the common flux which gets, by this approach, its identity, therefore their influence may be calculated with better precision.

The main argument of the Maxwellian approach was the claim that the parameters in the equations of the type (13) are fundamental, and that they may express the leakage via the coupling coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}}, \quad (19)$$

or the coefficient of total leakage

$$\sigma = 1 - k^2 = 1 - M^2 / (L_1 L_2). \quad (20)$$

And, really, it happens that the presentation with common and leakage fluxes fails already by simple three-coil transformers, when one gets a negative value for some leakage self-inductance.

This kind of discussion, more implicit than explicit, lasted during the entire eight year period. Arnold [11,15] uses the Maxwellian approach, but sees, in 1895, that still a difference should be made between the primary and secondary leakage, and defines the coefficients according to:

$$\begin{aligned}\sigma_1 &= M / L_1 = 1/v_1 \\ \sigma_2 &= M / L_2 = 1/v_2\end{aligned}\quad (21)$$

where v_1 and v_2 are Hopkins' leakage coefficients so the coefficient of total leakage becomes:

$$\sigma = 1 - \sigma_1\sigma_2. \quad (22)$$

Blondel appeared in a German magazine [17], claiming that only Hopkins's coefficients have a practical sense, showing that on his "fundamental" (vector) diagram of multiphase motor, published in a French magazine in 1893 [12]. Regarding the debate about the two approaches, it is interesting to note that Blondel mentions, in that same article, by the end, a theory of multi-phase motors of a "completely different kind", which was first given by Kapp for transformers, and then, together with Steinmetz, for multi-phase motors as well. It consists, according to Blondel, in the fact that the leakage of the primary and secondary circuits are represented by "two special self-inductances". The total fluxes may then be imagined as if coming from a common flux, and from fluxes which originate in those self-inductances. He does not criticize the theory of another kind, but shows that, in fact, here is the question of two different engineers' representations of the same phenomenon and proves the equivalency of two vector diagrams by equations of fluxes in one and in the other way. He still thinks that Hopkins's coefficients may be more easily understood and calculated, than the self-inductance.

Fig.3a shows the Blondel's phasor diagram of fluxes according to Maxwell's equations (13), but with separately marked "fictive" stator (index s) and rotor (index r) fluxes

$$\begin{aligned}\psi'_s &= Mi_s \\ \psi'_r &= Mi_r\end{aligned}\quad (23)$$

so that, using the Hopkins's leakage coefficients $v_s > 1$ and $v_r > 1$, they become:

$$\begin{aligned}\psi_s &= v_s\psi'_s + \psi'_r \\ \psi_r &= \psi'_s + v_r\psi'_r\end{aligned}\quad (24)$$

which is shown on the diagrams with two triangles with thicker lines. The thinner lines give the diagram according to the "theory of another kind" (blown up on Fig. 3b) with which Blondel establishes the connection by equations:

$$\begin{aligned}(v_r - 1)\psi'_r &= \Lambda_r i_r \\ (v_s - 1)\psi'_s &= \Lambda_s i_s\end{aligned}\quad (25)$$

As one can see, the diagrams at Fig. 3b include the common flux ψ_m , the scattering fluxes λ_s and λ_r and the magnetizing current i_m according to the equations (18).

The question of the treatment of the scattering fluxes and of the introduction of the common flux looks like only the question of the choice of engineers' *representation* of the phenomena in the machine, and not as a substantial question, or a question which is substantial for the calculation of a machine – even regardless of the fact that the phenomena take place in air medium, or with the existence of iron. That is, indeed, so, until one takes into account the special magnetic properties of iron medium, i.e. until one may neglect the phenomena of magnetic hysteresis and eddy currents, and of magnetic saturation. Hysteresis and eddy currents create additional losses in the machine, which often can not be neglected in the calculations of machines, and the non-linear relations between the flux and its excitation in various régimes of function may have a strong influence on the operational characteristics of a machine. Developed by theoreticians who were closely connected to practice, and supported by engineers-constructors, this new approach was soon generally accepted, although the discussions about it continued in the next century, as well.

It is interesting to note that even today, in some elementary textbooks, the notion of primary and secondary leakage is not included at all.

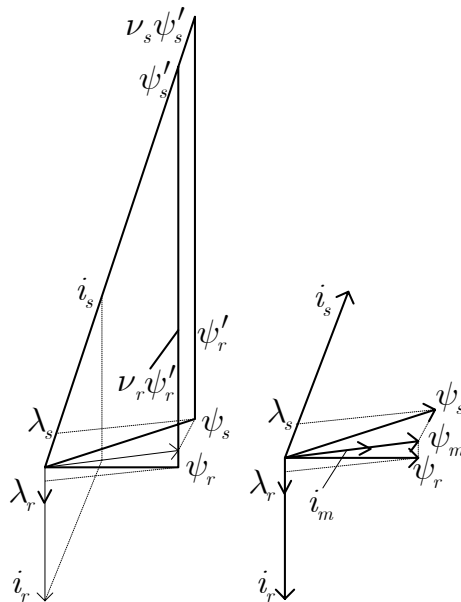


Fig. 3 – Phasory diagram of asynchronous motors
 a) with “fictive” fluxes; b) with common and leakage fluxes.

6 Iron

As early as in 1885-86, Kapp and Hopkinson, independently one from the other, elaborated a method for the calculation of direct current motors, which was based on the introduction of the notion of magnetic circuit with magnetic reluctances, analogous to electric circuits with resistances connected in series. They calculated the resulting flux taking into account the variable permeability, i.e. quantifying it using the experimentally obtained curves. Then, in 1887 [18], Kapp applied that to dampers and transformers, introducing, at the same time, the graphic method instead of differential equations. Steinmetz, who was already famous for his studies of the magnetic properties of iron, and especially of hysteresis, improved in 1891 the Kapp's theory of transformers, including the graphic methods of analysis and made a big step forward in the analysis of alternate currents in general, by introducing complex algebra [19]. In 1893, at the International Congress of Electricians in Chicago, he published the detailed theory of complex values [20], and the next year he formed the theory of asynchronous motors in the magazine of the American Institute of Electrical Engineers (AIEE) [21]. In 1895, Steinmetz, who was at that time an American engineer, but with German education, presented that same theory in the leading German magazine in a long article entitled "The Theory of Induction Motors" [22].

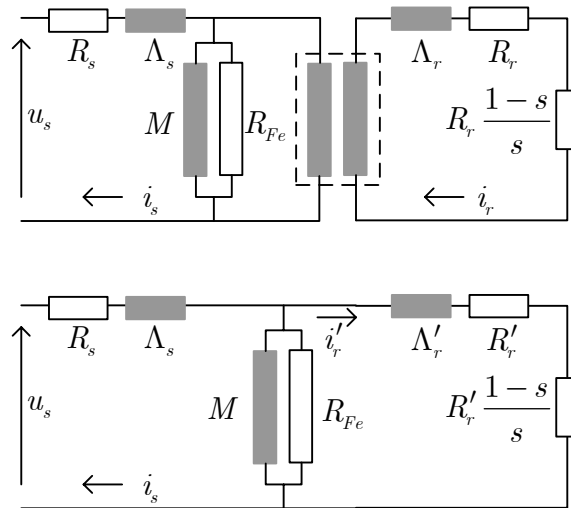


Fig. 4 – Equivalent schemes of asynchronous motor according to the analyses of Kapp, Steinmetz and Heyland (1894).

Regarding this, there was an interesting duel between Steinmetz and Pupin, published in the same magazine, in which Pupin opposed Steinmetz's deviation from Maxwell [23]. Even many years later, on a similar occasion, Pupin

remained persistent in his attitude, sharply criticizing the engineers for their deviation from Maxwell in their theories on asynchronous motors [6].

So, Kapp and Steinmetz, introducing scattering inductances and “primary admittance”, which represents not only the magnetizing current but the losses in iron as well, created a picture of the phenomena, which is presently usually represented by the equivalent T-scheme, and which is, except in the secondary circuit, the same as the scheme of two-winding transformer (Fig. 4).

7 Number of Wires

It has already been said that Nikola Tesla invented the polyphase asynchronous motor in the framework of his research of commutatorless system of production, transmission and use of electric power. However, it happened that the motor itself was immediately accepted as a serious alternative to the motor with direct current, but not the polyphase system of power transmission. The majority of engineers and theoreticians were of the opinion that bringing the supply of electric power with more wires than two represented a serious shortcoming of the asynchronous motor.

In the mentioned pioneer work by Hutin and Leblanc from 1891 [8] the multiphase system is not mentioned at all; the work presented an analysis of a motor with single phase supply and auxiliary phase through a capacitor. That is why one of the substantial conclusions of their analysis is that the frequency of supply should be as high as possible. (they worked with 75 Hz, for their experimental dynamo could not produce more). In one of the first works of Behn-Eschenburg in which, as we already saw, the exact formula of the dependence of torque on the velocity [13] was presented for the first time, three motors are equally treated: the polyphase motor, the single phase motor and the asynchronous motor of a special type, to which an additional collector with short-circuited brushes was added, in order to assure the start. In the entire analysis, it is understood that the single phase motor is the objective, while attention was paid to the polyphase motor only in order to find a solution for the start of single phase motor. Only in a later article, next year [24] the same author reaches a conclusion which strongly advocates the polyphase motor: not only by analysis, but also experimentally he got convinced that the three-phase motor was 3-4 times more capable (“leistungsfähig”) than that same motor when it is connected to function as single phased. Görges, too, brings at the end of his broad presentation of the theory of asynchronous motor [25], as the main conclusion, that the single phase motor is much inferior in comparison with the three-phase one. Tesla himself submitted, in the period 1888-1891, a number of patents on single phase motor, and Steinmetz patented in America in 1894 his “monocycle system”, which represents a compromise between the single phase

and the three-phase transmission: two wires for lighting, the third one is added only there where there are motors.

It seems that the most persistent advocate of the polyphase motor, from the very beginning was Dolivo-Dobrowolsky and the firm AEG in which he worked. He was of the opinion, as it was already mentioned, that the number of phases should be as big as possible in order to have less pulsations in the torque. Together with the Swiss Braun, from the firm Oerlikon, Dolivo-Dobrowolsky introduced important novelties in the construction of the three-phase motor, so that their motors became the models for production, not only in Europe, but in America as well. It is interesting to note that Braun, when he left Oerlikon and established his own firm (later on the well known firm Braun-Boveri) abandoned the polyphase system, while Dolivo-Dobrowolsky and the firm AEG continued to work on the three-phase system.

8 Rounding

The rounding of the theory of steady states of asynchronous motors, as we know it at present, was brought by the following years 1894, 1895 and 1896. Beside the already mentioned Steinmetz's contribution regarding the accounting for the losses in iron, those years were important because of the circular diagram which showed to be very practical for the calculation of asynchronous motors and which was developed, independently from one another Heyland and Berend. Heyland, in his article in 1894 [20] first reduces the secondary to the primary according to the relationship of the windings. Then he defined the leakage inductances in the following way: L_1 , L_2 and M are of the same order of magnitude, which means that for $L_1 L_2 > M^2$, $L_2 > M$, and let $L_1 = M + A_1$, $L_2 = M + A_2$. "There, L_1 and L_2 may be conditionally imagined as the number of lines of force, which each of the conductors must develop before they are cut with the other conductor." He first developed a diagram for a simple transformer, then for a transformer with rotating field, which is equivalent to the standstill of the motor. Finally, he starts the motor and establishes that the only difference lies in the fact that by transformers ω_1 in the quotient ω_1/R_2 is constant, and R_2 variable, while by motors ω_2 in ω_2/R_2 is variable, and R_2 constant. If we put $\omega_2 = \omega_1$, it comes out that the motor behaves in everything like a transformer with rotor resistance R_2/s . This is quite near to the present equivalent scheme of motors. However, the losses due to hysteresis and eddy currents are neglected.

In 1895, Heyland supplemented his ideas as an answer to the objections raised against his article [27], and in 1896, Berend presented his phasor diagram of polyphase motor, which is based on the diagram of transformers from Kapp's

book [28] which was published at that time. Then, out of that diagram he derived the circular diagram. Finally, Heyland, in his article in 1896 [29] presented a quite broad theory of phasor and circular diagrams of motors, while one of the pioneers of the theory of asynchronous motors, Behn-Eschenburg, appeared again with the explanation of that theory, but in a new form, adapted to constructors [30].

The explanation of Tesla's discovery lasted, therefore, in that phase, some ten years. During that time the theories of asynchronous motors were developed; those theories gave, as end result, in the form of formulas, the main dependences between physical values which characterized the phenomena in the machines in steady state. That knowledge gave the answers to the question linked to the construction of better and cheaper motors, which was very important for the manufacturers in the sharp competition which soon appeared. Besides, the engineering minded researchers developed various kinds of plastic presentation of physical phenomena in the motor as a replacement for analytical theories, which may include and express everything, but in its introvert and not enough operational way. These representations, like the application of complex algebra, the phasor diagrams, the equivalent scheme and the circular diagram, gave not only a pictorial and qualitative presentation of the phenomena, which enabled easier thinking, but, besides, gave data of quantitative character with direct practical application.

8 The General Theory of Machines

As we already saw, all substantial properties of asynchronous motors were studied within the first ten years after its birth. The theory, which could not only explain their behavior, but gave instructions for their construction as well, was basically rounded. The big manufacturers of electric machines, like Oerlikon and AEG in Europe, and Westinghouse and General Electric in the USA, which started already in 1890 the construction of the first motors of the new kind engaged themselves, together with many other firms in mutual competition at the new market. Tesla's polyphase system for the production and transmission of electric power was generally accepted, and the asynchronous motor started to overtake all fields of application which needed the constant velocity of the shaft, where, until that time, the direct current machines had been dominant. Besides, the field of application of electric drives in general was very much expanded, for the industry, thanks to the smaller price and robustness of the new motors decided more easily to replace their old drives with centralized belt power transmission with individual machines.

Regarding the construction, the progress in the construction of motors in the first sixty years of the twentieth century, was not any more revolutionary, but gradual. Still step by step, mostly thanks to the improvement in insulation,

ventilation and bearings, the motors of the ‘sixties of the twentieth century became more than ten times more powerful in comparison with those from the end of the nineteenth century. This gradual progress is illustrated on Fig. 5, which shows how, with the given overall size, the power of the asynchronous motor increased in the firm Westinghouse during that period [31]. During that period, by the beginning of the last century, the introduction of the double-caged rotor and of the rotor with deep grooves, was of special importance, for it solved the problem of good starting torque in motors with short-circuited rotors.

Of course, these improvements were followed by corresponding theoretical research. Let us mention the results of the studies of higher spatial harmonics in the rotation field, which influenced the ways of winding of the stators, the design of the form of the slots, the selection of the number of slots, etc., and the theoretical research of Field [32], Emde [33] and others, who dealt with the so-called current suppression, i.e. with the additional losses in copper caused by eddy currents in the conductors of the rotor and influenced not only the increase of the useful power of the motor, but the way of design of double-cage rotors and rotors with deep slots as well.

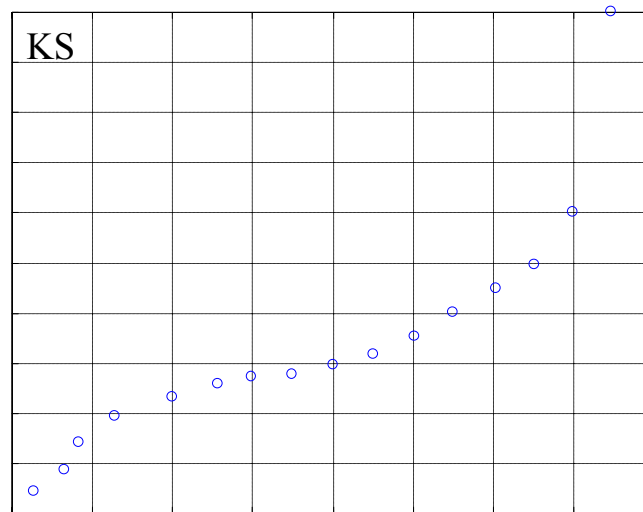


Fig. 5 – *The increase of the power of the motor with the same overall size during years.*

However, it is interesting to note that until the ‘sixties of the last century, there was no special interest in the study of the electrodynamic properties of asynchronous motors and in transitional electromagnetic processes in general. That is quite understandable, since that motor was adopted as a motor with constant speed, i.e. rigid and uncontrollable, and people did not count on it for the use in regulated drives. Although there appeared technical solutions which

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solved that problem, too, like the Scherbius and Kramner cascades, the motor with direct current remained dominant for a long time for regulated drives.

The progress of the theory of asynchronous motors, especially regarding the transition states, came from another side, by the beginning of the 'thirties, via a similar machine, also with rotating field – the synchronous generator. Stimulated by the practical problems of the development of polyphase networks, with an increasing number of generators in parallel operation, and primarily by the problem of stability, the studies of the transition states of those machines became increasingly important. It became soon clear that all machines with rotating field may be treated in the same way, and that was the beginning of the development of the general theory of electric machines. That started, more in a hidden way than explicitly, the analysis of the transition phenomena in asynchronous machines.

In 1929, R.H. Park explained, in America, the theory of synchronous machines “with two reactions” [35] and in 1933, André Blondel published in France the book on “general methods of study of sinusoidal currents” [34]. The basis of those theories was the so-called breakdown of the rotating field into two orthogonal components – longitudinal and transversal – in order to solve in that way the question of different magnetic reluctances on the ways of the flux along the poles and between the poles by machines with salient poles. In fact, that was the beginning of the idea of the matrix transformations in machines in general, and in asynchronous machines as well, although they do not have salient poles. The set of equations which describe the phenomena in the machines (the mathematical model) became simpler also thanks to the “transformation of coordinates” to the coordinate system linked to the side with salient poles (rotor by synchronous machines) which eliminate the trigonometric functions of the angle of the rotor, i.e. time, and the differential equations in the model, although still non-linear, become stationary. Moreover, in the conditions of constant speed, they become linear as well, and thanks to the application of the Laplace transformation, it is possible to derive the transition functions and to study all electromagnetic transition processes in the machines in the way which is known in the theory of regulated systems. In 1938, Gabriel Kron introduced the tensorial, in fact the matrix calculation in the treatment of the phenomena in machines [36] and supplemented later that theory with a broad book on equivalent schemes, 1951. [37] The first books which dealt in more detail with the transition phenomena by synchronous machines were published by the American Concordia 1951 [38] and the Swiss Laible 1952 [39].

Simultaneously with these works, there appeared in a completely different field a development which contributed unintentionally to the theory of electric machines and especially to the study of the transition states. That was the development in the field of polyphase electric grids and, although it looks quite

paradoxical, it was the progress in the analysis of steady, not transition states. In 1918, Fortescue proposed the “method of symmetric coordinates” for the solution of the problem of non-symmetrical steady states, which was of special importance for the practice for the calculation of non-symmetric short-circuits in the grids [41]. The method consists in the breakdown of the non-symmetric system of three-phase currents or voltages represented by complex values (phasors) into two symmetric and one, so-called zero system, in order to treat each new system separately in the known way, with the superpositioning of the results at the end. Later on, another method with the same objective, the breakdown in alphabeta components by Edith Clark, was added to that one [42]. This breakdown, or, in the language of matrix calculus, transformation of non-symmetric phasors, e.g. those which represent a set of three-phase currents

$$\bar{\mathbf{I}}_{abc} = [\bar{I}_a \quad \bar{I}_b \quad \bar{I}_c]^T \quad (26)$$

is done with the formula

$$\bar{\mathbf{I}}_{0di} = \mathbf{F}^+ \bar{\mathbf{I}}_{abc}, \quad (27)$$

where

$$\bar{\mathbf{I}}_{0di} = [\bar{I}_0 \quad \bar{I}_d \quad \bar{I}_i]^T \quad (28)$$

is a new set with phasors which represent the so-called direct (d) and inverse (i) symmetric system of currents and of zero component (0), and

$$\mathbf{F}^+ = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & \bar{a} \\ 1 & \bar{a} & a \end{bmatrix}, a = e^{j2\pi/3}, \bar{a} = e^{-j2\pi/3} \quad (29)$$

is the matrix of the transformation. In the method of Edith Clark everything is identical, except that instead of \mathbf{F}^+ the transformation matrix reads

$$C^+ = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1/2 & -1/2 \\ 1 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}. \quad (30)$$

This matrix is nothing else than the real variant of the previous matrix, for it is obtained by the linear combination of the second and third kind (adding and subtracting).

The mentioned paradox lies in the fact that the same transformation matrices, intended for the solution of non-symmetric but steady states may be applied to the momentaneous values of a set of polyphase values (e.g. on three-phase currents, voltages or fluxes), whether in steady or transition processes,

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symmetric or non-symmetric states, sinusoidal or non-sinusoidal forms of waves, with excellent effects: the mathematical model of the machine is substantially simplified (the effect of unharnessing), even the order of system of differential equations is reduced in most of the cases (the effect of reduction of order). Moreover, since the second and third kind of the transformation matrix F+ are mutually conjugated, the set consisting of the values of all phases, e. g. the momentaneous values of three three-phase currents i_a , i_b and i_c , may be expressed by one single algebraic transformation formula (under the assumption that there are no zero components, which is the most frequent case in the analysis of machines) which reads:

$$i = \frac{1}{\sqrt{3}}(i_a + ai_b + \bar{a}i_c)e^{-j\theta_z}, \quad (31)$$

or, in general for any number of phases q and currents i_1, i_2, \dots, i_q :

$$i = \frac{e^{-j\theta_z}}{\sqrt{q}} \sum_{k=1}^q a^{k-1} i_k, \quad a = e^{j2\pi/q}, \quad (32)$$

-of course, with appropriate transformation formulae for voltages, fluxes and parameters of the machine. By selecting the angle theta in fact we select the already mentioned coordinate system, i.e. we select a suitable rotation of coordinates.

In that way, you get one single, although complex, value (here i) – according to the modern terminology a *space vector* or *polyphasor* – which represents at every moment the entire polyphase system with, in this case q currents. The formulae like (31) and (32) represent the key for the approach to all transformations, including the mentioned Blondel's and Park's "breakdown" to longitudinal and transversal components and, as it was said, they make possible the formation of simple mathematical models of all polyphase machines [40].

Until the beginning of the sixties, the general theory of electric machines at the base of matrix transformations was very much rounded and presented systematically in several books, the most important among them being the books by the Americans Lyon [43], White and Woodson [44], Ku [45], Seely [46], and of the Hungarians Kovacs and Racz [47]. Thus, the theory was ready in advance for the new challenge to be brought by the development of technology in the 'sixties to the asynchronous motor – and that is the appearance of power electronics.

9 Renaissance

By the beginning of the sixties, there started a new era in the application of the asynchronous motor [4]. The appearance of powerful semi-conductor switches – thyristors – opened the way to much more economical and technologically better achievements of invertors – converters of direct current into alternate current – and, thus, to the converters of frequency. By changes of frequency the velocity of asynchronous motors may be adjusted continuously and without important increase of losses. As the price of those converters dropped down, the application of asynchronous motors in the drives with variable speed increased – in the field in which the direct current machines were dominant. And since in that way the asynchronous motor became an element of the systems of automatic control, the interest in mathematical description of phenomena in it increased as well. The dynamic mathematical model, as an integral part of the general theory of machines was, as we already saw, ready, but still there appeared a series of theoretical articles mainly with the objective of its inclusion in the theory of control and adaptation to computer processing.

However, an important breakthrough in the theoretical sense was the discovery of the so-called vector control by the beginning of the seventies [49]. Namely, it appeared that the asynchronous motor as an element of regulated system was in dynamic respect, much worse than the machines with direct current. From the standpoint of control, both machines had two input variables each, which could influence two internal values which are of substantial importance for the output mechanical effect, and these are the exciting flux and the current, whose product gives the torque. In direct current machines, it is possible to influence independently these two factors – influence the flux via exciting current, and current via voltage or even directly. The point is in the fact that the control of the flux in every machine is slow because of the big magnetic inertia, but that slowness is avoided in direct current by adjusting the flux once to an appropriate value, and the further control of the torque is performed by the current with very small or non-existing lag. According to the general theory of machines, the asynchronous machine must also have such two inputs, and it has it, really, but neither of them is directly accessible. The consequence of that is that in the control via any of the two accessible values (e.g. via voltage or frequency) the magnetic inertia must participate, therefore the reaction of the machine to the change of any input is slow. The solution was found in the mathematical calculation of the inaccessible internal values out of the values which are measurable, like the momentaneous velocity, the voltage and the current in the motor.

This way of control of asynchronous motors, later called vector control, was immediately adopted by the firm Siemens which put on the market a product

called “transvector” but still, fifteen years had to elapse until the broad acceptance of the vector control. The reason for that is that the mentioned calculation included the multiplication and deriving of trigonometrical functions in realtime, which could not be done at that time enough precisely and efficiently by the analogue technology of that time. Only with the introduction of the microprocessor technology into the regulated driving systems, quite recently, when the sixteen bite processors became economically accessible, vector control got its full swing.

During the last dozen of years the theory of the asynchronous motors was again the object of interest of many researchers and engineers, almost like in the first years after Tesla’s discovery. Already now, hundreds of works have been published in professional magazines or presented at conferences, while the digitally regulated drives with vector regulation are already present in the catalogues of many manufacturers. So the asynchronous motor got the chance to press out the motors with direct current from the last field of application in which they persistently remained until recently – from the drives with high performance which require quick reaction to regulation requests [48].

After the starting ten years of development of the theory, and the quick conquest of a broad field of application in the drives with constant speed and relative stagnation during a number of decades, Tesla’s motor becomes, in our time again, the inspiration for researchers and engineers. They will give it a new life and make it attractive in a broad area of applications, starting with simple industrial drives, until the highest quality velocity and position servo systems.

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