Robust State Feedback Controller Design of STATCOM Using Chaotic Optimization Algorithm

Amin Safari\textsuperscript{1}, Hossein Shayeghi\textsuperscript{2}, Heidar Ali Shayanfar\textsuperscript{3}

Abstract: In this paper, a new design technique for the design of robust state feedback controller for static synchronous compensator (STATCOM) using Chaotic Optimization Algorithm (COA) is presented. The design is formulated as an optimization problem which is solved by the COA. Since chaotic planning enjoys reliability, ergodicity and stochastic feature, the proposed technique presents chaos mapping using Lozi map chaotic sequences which increases its convergence rate. To ensure the robustness of the proposed damping controller, the design process takes into account a wide range of operating conditions and system configurations. The simulation results reveal that the proposed controller has an excellent capability in damping power system low frequency oscillations and enhances greatly the dynamic stability of the power systems. Moreover, the system performance analysis under different operating conditions shows that the phase based controller is superior compare to the magnitude based controller.

Keywords: STATCOM, State feedback damping controller, Chaotic optimization algorithm, Low frequency oscillations.

Nomenclature

\begin{center}
\begin{tabular}{ll}
COA & Chaotic Optimization Algorithm \\
STATCOM & Static Synchronous Compensator \\
DC & Direct Current \\
SVC & Static Var Compensator \\
$E'_q$ & Internal voltage behind transient reactance \\
$T_A$ & Regulator time constant \\
$E_{fd}$ & Equivalent excitation voltage \\
$T'_{d0}$ & Time constant of excitation circuit \\
FACTS & Flexible Alternating Current Transmission Systems \\
\end{tabular}
\end{center}

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1 Introduction

Intensive progress in power electronics has enabled the application of Flexible AC Transmission System (FACTS) devices in high voltage transmission networks. The main objective of using FACTS devices are normally the steady-state control of a power system, but due to their fast response, FACTS can also be used for power system stability enhancement by improving damping of the power swings [1]. Through the modulation of bus voltage, phase shift between buses, and transmission line reactance, FACTS devices can cause a substantial increase in power transfer limits during the steady-state conditions.

The STATCOM is based on the principle that a voltage-source inverter generates a controllable AC voltage source behind a transformer-leakage reactance so that the voltage difference across the reactance produces active and reactive power exchange between the STATCOM and the transmission network [2, 3]. It is reported that the STATCOM can offer a number of performance advantages for reactive power control applications over the conventional approaches, such as Static Var Compensators (SVC), because of its greater reactive current output capability at depressed voltage, faster response, better
control stability, lower harmonics and smaller size, etc. [2]. Several trials have been reported in the literature to dynamic models of the STATCOM in order to design suitable controllers for AC, DC voltage and damping controls. Wang [3] presents the establishment of the linearized Phillips-Heffron model of a power system installed with a STATCOM. Further, no effort seems to have been made to identify the most suitable STATCOM control parameter, in order to arrive at a robust damping controller. Fuzzy-logic-based controllers have, for example, been used for controlling a STATCOM [4]. The performance of such controllers can further be improved by adaptively updating their parameters. Also, although by using the robust control methods [5, 6], the uncertainties are directly introduced to the synthesis, but due to the large model order of the power systems the resulting controller order will be very large in general, which is not feasible because of the computational difficulties and economical expenses in implementation.

In this paper, COA is used for robust tuning of the STATCOM based state feedback damping controller in order to enhance the damping of the power systems low frequency oscillations. Chaos is a kind of characteristic of nonlinear systems which is a bounded unstable dynamic behavior, which exhibits sensitive dependence on the initial conditions and includes infinite unstable periodic motions. The COA is based on the ergodicity, stochastic properties and regularity of the chaos. It is not like some stochastic optimization algorithms that escape from the local minima by accepting some bad solutions according to a certain probability, but COA searches on the regularity of the chaotic motion to escape from the local minima [7-13].

A new approach for the optimal design of state feedback gains for the STATCOM damping controller is investigated in this paper. The problem of robust state feedback damping controller design for STATCOM is formulated as an optimization problem and COA is used to solve it. Since only local states are used as the inputs of each controller, the optimal design of the controller can be accomplished. The effectiveness of the proposed controller is demonstrated through nonlinear time-domain simulation studies to damp low frequency oscillations under different operating conditions. Results evaluation show that the proposed damping controller achieves good robust performance for a wide range of operating conditions and disturbances.

2 System Model with STATCOM

To study the new control strategy for the STATCOM, the single machine infinite bus power presented in Fig. 1 is considered for the transient stability simulations.
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The synchronous generator is delivering power to the infinite-bus through a double circuit transmission line and a STATCOM. The system data is given in the Appendix. The system consists of a step down transformer with a leakage reactance XSDT, a three phase GTO-based voltage source converter, and a dc capacitor [3].

![Diagram](image)

**Fig. 1 – SMIB power system equipped with STATCOM.**

### 2.1 Power system nonlinear model with STATCOM

The VSC generates a controllable AC voltage source \( v_o(t) = V_0 \sin(\omega t - \varphi) \) behind the leakage reactance. The voltage difference between the STATCOM bus AC voltage, \( v_L(t) \) and \( v_o(t) \) produces active and reactive power exchange between the STATCOM and the power system, which can be controlled by adjusting the magnitude \( V_0 \) and the phase angle \( \varphi \). The dynamic relation between the capacitor voltage and current in the STATCOM circuit are asserted as [3]:

\[
\begin{align*}
\dot{I}_{Lo} &= I_{Lo} + j I_{Lq}, \\
V_o &= c V_{dc} (\cos \varphi + j \sin \varphi) = c V_{dc} e^{j \varphi}, \\
\dot{V}_{dc} &= \frac{I_{dc}}{C_{dc}} = \frac{c}{C_{dc}} (I_{Lod} \cos \varphi + I_{Lq} \sin \varphi),
\end{align*}
\]

where, for the PWM inverter \( c = mk \) and \( k \) is the ratio between AC and DC voltage depending on the inverter structure, \( m \) and \( c \) are the modulation ratio and magnitude defined by the PWM. The \( C_{dc} \) is the dc capacitor value and \( I_{d0} \) is the capacitor current while \( i_{L0d} \) and \( i_{L0q} \) are the d and q components of the STATCOM current, respectively.

The dynamics of the generator and the excitation system are expressed through a fourth order model given as [5, 8, 16]:

\[
\begin{align*}
\dot{\delta} &= \omega_0 (\omega - 1), \\
\dot{\omega} &= \frac{(P_m - P_e - D\Delta \omega)}{M}
\end{align*}
\]
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$$E'_q = (-E_q + E_{fd}) / T_{d0} \tag{6}$$

$$E_{fd} = (-E_{fd} + K_a (V_{ref} - V_t)) / T_a \tag{7}$$

The expressions for the d-q axes currents in the transmission line and STATCOM are given as follows [5]:

$$I_{ld} = \left(1 + \frac{X_{LB}}{X_{SDT}}\right) e'_q \frac{X_{LB}}{X_{SDT}} - m V_{dc} \sin \varphi - V_b \cos \varphi$$

$$X_{dL} + X_{LB} + \frac{X_{dL}}{X_{LB}} \left(1 + \frac{X_{LB}}{X_{SDT}}\right) x'_d \tag{8}$$

$$I_{ldq} = \frac{X_{LB}}{X_{SDT}} m V_{dc} \cos \varphi + V_b \sin \varphi$$

$$X_{dL} + X_{LB} + \frac{X_{dL}}{X_{LB}} \left(1 + \frac{X_{LB}}{X_{SDT}}\right) x_q \tag{9}$$

$$I_{lod} = e'_q - (x'_d + X_{dL}) I_{dlq} - m V_{dc} \sin \varphi$$

$$X_{SDT} \tag{10}$$

$$I_{loq} = \frac{m V_{dc} \cos \varphi - (x'_d + X_{dL}) I_{dlq}}{X_{SDT}} \tag{11}$$

where, $X_{dL} = X_T + X_L / 2$, $X_{LB} = X_L / 2$.

The $X_T$, $x'_d$ and $x_q$ are the transmission line reactance, d-axis transient reactance, and q-axis reactance, respectively.

### 2.2 Power system linearized model

A linear dynamic model is obtained by linearizing the nonlinear model around an operating point. The linearized model of the power system as shown in Fig. 1 is given as:

$$\Delta \dot{\delta} = \omega_0 \Delta \omega, \tag{12}$$

$$\Delta \dot{\omega} = (\Delta P_e - D \Delta \omega) / M, \tag{13}$$

$$\Delta \dot{E}'_q = (\Delta E_q + \Delta E_{fd}) / T'_{d0}, \tag{14}$$

$$\Delta \dot{E}_{fd} = (K_a (\Delta v_{ref} - \Delta v) - \Delta E_{fd}) / T_a, \tag{15}$$

$$\Delta \dot{v}_{dc} = K_g \Delta \delta + K_v J L \Delta E'_q - K_v J L E_{dc} + K_v J L \Delta c + K_{deq} \Delta \varphi, \tag{16}$$

$$\Delta P_e = K_1 \Delta \delta + K_2 \Delta E'_q + K_{pdc} \Delta v_{dc} + K_{pc} \Delta c + K_{p\varphi} \Delta \varphi, \tag{17}$$
\[ \Delta E'_q = K_4 \Delta \delta + K_3 \Delta E'_q + K_{qdc} \Delta v_{dc} + K_q \Delta c + K_{q\phi} \Delta \varphi, \]  
\[ \Delta V_t = K_5 \Delta \delta + K_6 \Delta E'_q + K_{vdc} \Delta v_{dc} + K_v \Delta c + K_{v\phi} \Delta \varphi, \]  

where, \( K_1, K_2, \ldots, K_9, K_{pu}, K_{qu} \) and \( K_{vu} \) are linearization constants. The state-space model of the power system is given by:

\[ \dot{x} = Ax + Bu, \]  

\[ x = [\Delta \delta \quad \Delta \omega \quad \Delta E'_q \quad \Delta E_{fd} \quad \Delta v_{dc}]^T; \quad u = [\Delta c \quad \Delta \varphi]^T, \]

\[
A = \begin{bmatrix}
0 & w_0 & 0 & 0 & 0 \\
\frac{-K_1}{M} & 0 & \frac{-K_2}{M} & 0 & \frac{-K_{pdc}}{M} \\
\frac{K_4}{T_d} & 0 & \frac{K_3}{T_d} & -1 & \frac{-K_{qdc}}{M} \\
\frac{-K_4 K_5}{T_d} & 0 & \frac{K_A K_6}{T_d} & -1 & \frac{-K_{vdc}}{M} \\
K_7 & 0 & K_8 & 0 & -K_9 \\
\end{bmatrix} \quad ; \quad B = \begin{bmatrix}
0 & 0 \\
\frac{-K_{pc}}{M} & \frac{-K_{p\phi}}{M} \\
\frac{K_{qc}}{T_d} & \frac{-K_{q\phi}}{T_d} \\
\frac{-K_{vc}}{T_d} & \frac{-K_{v\phi}}{T_d} \\
\frac{K_{dc}}{T_d} & \frac{-K_{d\phi}}{T_d} \\
\end{bmatrix}.
\]

The block diagram of the linearized model of the SMIB power system with the STATCOM is shown in Fig. 2.

Fig. 2 – Modified Heffron–Phillips transfer function model.
2.3 State feedback damping controller for the STATCOM

The state feedback based damping controller is designed to produce an electrical torque in phase with the speed deviation according to the phase compensation method. A power system can be modeled by a set of nonlinear differential equations as:

\[ \dot{X} = f(X, U), \quad (21) \]

When a power system operates at one operating point, the system is linearized. The state-space model of power system is given by:

\[ \dot{x} = Ax + Bu, \quad (22) \]

where the input vector \( u \) is composed of the control vector \( U_s \) and the disturbance vector \( U_d \), both \( m \times 1 \) vectors; \( x \) is an \( n \times 1 \) state vector; \( A \) is an \( n \times n \) plant matrix of the open-loop system and \( B \) is the \( n \times m \) input matrix; \( n \) and \( m \) are the number of state variables and control signals, respectively. In this paper \( C \) and \( \varphi \) are modulated in order to use it for the state feedback controller design. The following control input vector is defined:

\[ U_s = Kx, \quad (23) \]

where, \( K \) is the feedback gain matrix with appropriate dimensions. Applying (23) to (22):

\[ \dot{x} = (A + BK)x + Bu_d, \quad (24) \]

\[ A_c = A + BK, \quad (25) \]

where, \( A_c \) is a the closed-loop system matrix. By properly choosing the feedback gain \( K \), the eigenvalues of the closed-loop matrix \( A_c \) are moved to the left-hand side of the complex plane and the desired performance of the controller can be achieved. Thus, the remaining problem in the design of the output feedback controller is the selection of \( K \) to achieve the required objectives. The control objective is to increase the damping of the critical modes to the desired level.

3 Chaotic Optimization Algorithm

Chaos often exists in nonlinear systems. It is a kind of highly unstable motion of the deterministic systems in the finite phase space. An essential feature of the chaotic systems is that the small changes in the parameters or the starting values for the data lead to vastly different future behaviors, such as stable fixed points, periodic oscillations, bifurcations, and ergodicity [9]. This sensitive dependence on the initial conditions is generally exhibited by the
systems containing multiple elements with nonlinear interactions, particularly when the system is forced and dissipative. Sensitive dependence on the initial conditions is not only observed in the complex systems, but even in the simplest logistic equation [10]. The application of the chaotic sequences can be an interesting alternative to provide the search diversity in an optimization procedure. Due to the non-repetition of the chaos, it can carry out the overall searches at higher speeds than the stochastic ergodic searches which depend on the probabilities. The basic process of the chaos optimization algorithm generally includes two major steps. Firstly, defines a chaotic sequences generator based on the Lozi map. Generates a sequence of chaotic points and map it to a sequence of the design points in the original design space. Then, calculate the objective function with respect to the generated design points and chooses the point with the minimum objective function as the current optimum. Secondly, the current optimum is assumed to be close to the global optimum after certain iterations and it is viewed as the center with a little chaotic perturbation and the global optimum is obtained through the fine search. Repeat the above two steps until some specified convergence criterion is satisfied and then the global optimum is obtained [12]. This chaotic map involves also non-differentiable functions which make the modeling of the associate time series difficult. The Lozi map is given by [9, 14]:

$$ y_i(k) = 1 - a \times |y_i(k-1)| + y(k-1), \quad (26) $$

$$ y(k) = b \times y_i(k-1), \quad (27) $$

$$ z(k) = \frac{y(k) - \alpha}{\beta - \alpha}, \quad (28) $$

$$ x_i(k) = x_i + \lambda \times z_i(k) \times |U_i - x_i|, \quad r \leq 0.5, \quad (29) $$

$$ x_i(k) = x_i - \lambda \times z_i(k) \times |x_i - L_i|, \quad r > 0.5 $$

where, \( k \) is the iteration number. The values of \( y \) are normalized in the range [0, 1] to each decision variable in the n-dimensional space of optimization problem. Therefore, \( y_i \in [-0.6418, 0.6716] \) and \( [\alpha, \beta] = (-0.6418, 0.6716) \). The parameters used in this work are \( a = 1.7 \) and \( b = 0.5 \), as proposed by [11]. Many, unconstrained optimization problems with continuous variables can be formulated as the following functional optimization problem:

Find \( \mathbf{X} \) to minimize \( f(\mathbf{X}) \), \( \mathbf{X} = [x_1, x_2, \ldots, x_n] \),

where, \( f \) is the objective function, and \( \mathbf{X} \) is the decision solution vector consisting of the \( n \) variables \( x_i \), bounded by lower (\( L_i \)) and upper limits (\( U_i \))
Fig. 3 shows the flowchart of the proposed chaotic search procedure based on the Lozi map [9].

$M_g$ and $M_L$ are maximum number of iterations of the chaotic global search and maximum number of iterations of the chaotic local search, respectively. In this paper $\lambda$ is the step size in chaotic local search and linearly decreases from 0.1 to 0.01. Also, $\overline{f}$ and $\overline{X}$ are the best objective function and the best solution for the state feedback controller gains from the current run of the chaotic search, respectively.

4 Design of the State Feedback Controllers Using COA

In the proposed method, we must tune the STATCOM state feedback controller gains to improve the overall system dynamic stability in a robust way under different operating conditions and disturbances. A performance index based on the system dynamics after an impulse disturbance alternately occurs in the system is organized and used to form the objective function of the design
problem. In this study, an Integral of Time multiplied Absolute value of the Error (ITAE) is taken as the objective function. Since the operating conditions in power systems are often varied, a performance index for a wide range of operating points is defined as [8]:

\[ f = \sum_{i=1}^{N_p} \int_{t_{sim}}^{t_{sim}+t_{p}} |t \Delta \omega_i| \, dt, \]

(30)

where, \( t_{sim} \) is the time range of simulation and \( N_p \) is the total number of operating points for which the optimization is carried out. For the objective function calculation, the time-domain simulation of the power system model is carried out for the simulation period. The design problem can be formulated as the following constrained optimization problem, where the constraints are the controller parameters bounds:

Minimize \( f \) Subject to: \( G_{\min}^x \leq G_x \leq G_{\max}^x, \quad x = 1, ..., 5. \) (31)

The proposed approach employs COA to solve this optimization problem and search for an optimal set of output feedback controller gains. In any power system, the loading condition varies over a wide range. It is extremely essential to investigate the effect of variation of the loading condition on the dynamic performance of the system. In order to test the robustness of the state feedback controllers to wide variation in the loading condition, the uncertainty area for active and reactive power are selected as: \( 0.2 \leq P \leq 1.2 \) and \( 0.01 \leq Q \leq 0.4 \) pu [15]. The optimization of controller gains is carried out by evaluating the objective function as given in equation (30), which considers multiple of operating conditions. The operating conditions are considered as:

- Base case: \( P = 0.80 \) pu, \( Q = 0.2 \) pu and \( X_L = 0.4 \) pu (Nominal loading).
- Case 1: \( P = 0.2 \) pu, \( Q = 0.01 \) pu and \( X_L = 0.4 \) pu (Light loading).
- Case 2: \( P = 1.2 \) pu, \( Q = 0.4 \) and \( X_L = 0.4 \) pu (Heavy loading).
- Case 3: The 20% increase of line reactance \( X_L \) at nominal loading condition.
- Case 4: The 20% increase of line reactance \( X_L \) at heavy loading condition.

In order to acquire better performance, maximum number of iterations of chaotic global search and maximum number of iterations of chaotic local search are chosen as 500 and 100, respectively. Also, \( \lambda \) is the step size in chaotic local
search and linearly decreases from 0.1 to 0.01. It should be noted that the COA is run several times and then the optimal set of the controller parameters is selected. The final values of the optimized parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Controller Parameter</th>
<th>C Based Controller</th>
<th>$\phi$ Based Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>184.56</td>
<td>245.71</td>
</tr>
<tr>
<td>$G_2$</td>
<td>7.3446</td>
<td>1.0876</td>
</tr>
<tr>
<td>$G_3$</td>
<td>0.3445</td>
<td>0.0541</td>
</tr>
<tr>
<td>$G_4$</td>
<td>0.3876</td>
<td>0.8753</td>
</tr>
<tr>
<td>$G_5$</td>
<td>-1.565</td>
<td>0.9588</td>
</tr>
</tbody>
</table>

5 Nonlinear Time Domain Simulation

To investigate the power system performance, two classes of disturbances are studied. These classes are chosen to represent large, as well as small power system disturbances.

5.1 Response to small disturbance

To assess the performance of the proposed controllers, a disturbance of 0.3pu input torque is applied to the machine at $t = 1s$. The response of the system without controller is shown in Fig. 4.

The study is performed at three different operating conditions. The results are shown in Fig. 5. It can be seen that the COA based designed state feedback controller achieves good robust performance and enhance greatly the dynamic stability of the power systems.

5.2 Response to large disturbance

In this section, the performance of the proposed controllers under transient conditions is verified by applying a 6-cycle three-phase fault at $t = 1s$, in the middle of the $L_3$ transmission line. The fault is cleared by permanent tripping of the faulted line. The system response to this disturbance is shown in Fig. 6. It can be seen that the COA based designed controller achieves good robust performance at the wide range of the operating conditions. The system performance analysis under different operating conditions show that the $\phi$ based controller is superior compare to the C-based controller. Control signals
of the state feedback controllers are shown in Fig. 7 at nominal loading conditions. It can be concluded that the $\phi$-based controller provides much less control effort compared to the $C$-based controller.

**Fig. 4** – Response of the system without controller.

**Fig. 5a** – Dynamic responses for $\Delta \omega$ at nominal loading; Solid (COA based $\phi$ controller) and Dashed (COA based $C$ controller).

**Fig. 5b** – Dynamic responses for $\Delta \omega$ at light loading; Solid (COA based $\phi$ controller) and Dashed (COA based $C$ controller).
Fig. 5c – Dynamic responses for $\Delta\omega$ at heavy loading; Solid (COA based $\varphi$ controller) and Dashed (COA based $C$ controller).

Fig. 6a – Dynamic responses for $\Delta\omega$ at nominal loading; Solid (COA based $\varphi$ controller) and Dashed (COA based $C$ controller).

Fig. 6b – Dynamic responses for $\Delta\omega$ at light heavy loading; Solid (COA based $\varphi$ controller) and Dashed (COA based $C$ controller).
Fig. 6c – Dynamic responses for $\Delta \omega$ at heavy loading; Solid (COA based $\phi$ controller) and Dashed (COA based C controller).

Fig. 7 – Control signals of both stabilizers for a large disturbance at nominal loading; Solid (COA based $\phi$ controller) and Dashed (COA based C controller).

6 Conclusion

The chaotic optimization algorithm has been successfully applied to the design of the state feedback damping controller for the STATCOM. The design problem of the selecting state feedback controller parameters is converted into an optimization problem which is solved by a COA technique. Since chaotic mapping enjoys certainty, ergodicity and the stochastic property, the proposed algorithm increases its convergence rate and resulting precision that escape from the local minima. To ensure the robustness of the proposed damping controller, the design process takes into account a wide range of the operating conditions and system configurations. The effectiveness of the proposed controllers for improving transient stability performance of a power system is demonstrated by a single machine power system subjected to different severe disturbances. The non-linear time domain simulation results show the effectiveness of the proposed controllers and their ability to provide good damping of the low frequency oscillations.
7 Appendix
The nominal parameters of the system are listed in Table 2.

<table>
<thead>
<tr>
<th>Generator</th>
<th>$M = 8$ MJ/MVA</th>
<th>$T_{do} = 5.044$ s</th>
<th>$X_d = 1$ pu</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X'_q = 0.6$ pu</td>
<td>$X'_d = 0.3$ pu</td>
<td>$D = 0$</td>
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<tr>
<td>Excitation system</td>
<td>$K_q = 25$</td>
<td>$T_a = 0.05$s</td>
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<tr>
<td>Transformers</td>
<td>$X_f = 0.1$ pu</td>
<td>$X'_{SDT} = 0.1$ pu</td>
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<tr>
<td>DC link parameter</td>
<td>$V_{DC} = 1$ pu</td>
<td>$C_{DC} = 1$ pu</td>
<td></td>
</tr>
<tr>
<td>STATCOM parameter</td>
<td>$C = 0.25$</td>
<td>$\phi = 52^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_s = 1$</td>
<td>$T_s = 0.05$</td>
<td></td>
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</tbody>
</table>

8 References


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