A New Method to Minimize the Chattering Phenomenon in Sliding Mode Control Based on Intelligent Control for Induction Motor Drives

Ismail Bendaas¹, Farid Naceri¹

Abstract: This paper presents new method toward the design of hybrid control with sliding-mode (SMC) plus fuzzy logic control (FLC) for induction motors. As the variations of both control system parameters and operating conditions occur, the conventional control methods may not be satisfied further. Sliding mode control is robust with respect to both induction motor parameter variations and external disturbances. By embedding a fuzzy logic control into the sliding mode control, the chattering (torque-ripple) problem with varying parameters, which are the main disadvantage in sliding-mode control, can be suppressed. Simulation results of the proposed control theme present good dynamic and steady-state performances as compared to the classical SMC from aspects for torque-ripple minimization, the quick dynamic torque response and robustness to disturbance and variation of parameters.

Keywords: Induction motor; Sliding mode control, Fuzzy logic control, Fuzzy sliding mode control, Torque-ripple.

1 Introduction

Induction Motors (IM) are most suitable for industrial drives, because of their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, their control is a challenging task, because the rotor current, responsible for the torque production, is induced from the stator current and also contributes to net air-gap flux resulting in coupling between torque and flux [12]. The input-output linearization has been frequently used in nonlinear systems to find a direct relation between the system output and input in order to implement a control law. However, the complexity and the presence of high nonlinearities, in some cases do not have an exact compensation of these nonlinearities and thus obtain the desired tracking performance in the presence of external disturbances.

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Known by its robustness and simplicity of implementation, the sliding mode has been largely used to control a large class of nonlinear systems, [3, 4, 8, 11], [15]. To define a surface called sliding depending on system states in order to be attractive. The synthesized global control consists of two terms; the first allows the approach to this surface and the second maintain sliding along it towards the origin of the phase plane. The global control as well constructed ensures good tracking performance, rapid dynamic and short response time [16].

However, this control law represents a disadvantage in using the sign function in the control law to ensure the passage of the approach phase to the sliding mode. This gives rise to the phenomenon of chattering which consists of sudden and rapid variation of the control signal, consequently excite the high frequency of the process and it damages.

A fuzzy logic control has been a subject of active research since the work of Mamdani proposed in 1974 [9], the concept of FLC is to use the qualitative knowledge of a system to design a practical controller; it is generally applicable to plants that are ill-modelled, but qualitative knowledge of experienced operators available for design. It is particularly suitable for those systems with uncertain or complex dynamics. In general, a fuzzy control algorithm consists of a set of heuristic decision rules and can be regarded as a nonmathematical control algorithm, in contrast to a conventional feedback control algorithm [17], indeed, to remedy the disadvantage of the chattering phenomenon more works [4, 5, 6, 13], have been focused on the combination of sliding mode control with fuzzy logic control.

In this paper, we presented a new hybrid nonlinear control method which is based on sliding mode control and fuzzy logic method, sliding mode control approach is employed to design the induction motor speed and flux controllers. The dynamic decouple control has been accomplished under the condition that the parameter of stator resistance variants and the load torque is time variant. In order to reduce the undesired chattering phenomenon of signum function, the fuzzy control method is used, which can be used to design a new fuzzy switching function to replace the traditional sliding mode signum function, Finally, simulations and a comparison are presented to demonstrate the contribution of this approach.

2 Description model of an IM

The model of three-phase induction motor in the laboratory frame (α,β) is given by the following equations (1)-(5).

The voltage equations become:
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\[ V_{sa} = R_s i_{sa} + \frac{d\varphi_{sa}}{dt}, \]

\[ V_{sb} = R_s i_{sb} + \frac{d\varphi_{sb}}{dt}, \]

\[ V_{ra} = 0 = R_r i_{ra} + \frac{d\varphi_{ra}}{dt} + \omega\varphi_{r\beta}, \]

\[ V_{rb} = 0 = R_r i_{rb} + \frac{d\varphi_{rb}}{dt} - \omega\varphi_{ra}. \]  \hspace{1cm} (1)

The expression of electromagnetic torque and the movement for induction motor written as follows:

\[ T_e = p \frac{M}{L_r} (\varphi_{ra} i_{sb} - \varphi_{rb} i_{sa}), \]  \hspace{1cm} (2)

\[ J \frac{d\Omega}{dt} = T_e - T_L - f\Omega. \]  \hspace{1cm} (3)

The state model of the induction motor is a nonlinear system multivariable taking the following form:

\[ \dot{x}(t) = f(x) + g(x)u(t). \]  \hspace{1cm} (4)

The model of the induction motor, driven by (1), (2) and (3), is defined by the nonlinear system as follows:

\[
\begin{bmatrix}
\dot{i}_{sa} \\
\dot{i}_{sb} \\
\dot{\varphi}_{ra} \\
\dot{\varphi}_{rb} \\
\dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
-\gamma i_{sa} + \frac{K}{T_r} \varphi_{ra} + pK\omega \varphi_{r\beta} \\
-\gamma i_{sb} - pK\omega \varphi_{ra} + \frac{K}{T_r} \varphi_{r\beta} \\
\frac{M}{T_r} i_{sa} - \frac{1}{T_r} \varphi_{ra} - p\omega \varphi_{r\beta} \\
\frac{M}{T_r} i_{sb} + p\omega \varphi_{ra} - \frac{1}{T_r} \varphi_{r\beta} \\
\mu(\varphi_{ra} i_{sb} - \varphi_{rb} i_{sa}) - \frac{f}{J} \omega - \frac{T_L}{J}
\end{bmatrix} +
\begin{bmatrix}
\alpha & 0 \\
0 & \alpha \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
V_{sa} \\
V_{sb}
\end{bmatrix},
\]

\hspace{1cm} (5)

with:

\[ \sigma = 1 - \frac{M^2}{L_s L_r}, \quad K = \frac{M}{\sigma L_s L_r}, \quad \gamma = \frac{1}{\sigma L_s} \left( R_s + R_r \frac{M^2}{L_r^2} \right), \quad \mu = \frac{pM}{JL_r}, \quad \alpha = \frac{1}{\sigma L_s}, \]
3 Basic Concepts of VSC

A Sliding Mode Controller (SMC) is a Variable Structure Controller (VSC) [18]. A variable structure system is characterized by the choice of a function and a logic switching. This choice will switch at any time between the different structures, combining the useful properties of each of these structures in order to have the desired behavior of the system. Consider the system described by (4) [2, 4, 10].

The SMC design consists in achieving the following steps:

1) Design a switching manifold $s$ in the state space to represent a desired system dynamics, which is of lower order than the dimension of the given plant; $s$ is defined by:

$$s = \{ x \in \mathbb{R}^n : s(x) = 0 \},$$

where $s(x) \in \mathbb{R}^n$ is called the switching function.

2) Design a variable structure control:
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\[ u = \begin{cases} 
  u_{\text{max}}^+ (x), & \text{if } s(x) > 0, \\
  u_{\text{min}}^- (x), & \text{if } s(x) < 0, 
\end{cases} \tag{7} \]

such that any state \( x \) outside the switching surface is driven to reach this surface in finite time, that is, the condition \( s(x) = 0 \) is satisfied in finite time. Once on the switching surface, the sliding mode takes place, following the desired system dynamics. This procedure makes the VSC system globally asymptotically stable [15].

Equation (6) is a variety of sliding that divides the state space into two disjoint \( s(x) > 0 \) and \( s(x) < 0 \).

The switching logic is designed to force the trajectory to follow the surface switching. We then say that the trajectory of the system sliding along the surface switching \( s(x) = 0 \) is referred to the phenomenon of chattering [8, 13, 15].

So we are interested in calculating the equivalent control and then to calculate the attractive control defined in the state space (4), the control vector \( u \) is composed of two parameters \( u_{\text{eq}} \) and \( \Delta u \), we must find the analytical expression of the control \( u(t) \). We have:

\[ \dot{s}(x) = \frac{d}{dt} s = \frac{\partial s}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial s}{\partial x} \left\{ f(x,t) + g(x,t)u_{\text{eq}}(t) \right\} + \frac{\partial s}{\partial t} \left\{ g(x,t)\Delta u \right\}. \tag{8} \]

During the sliding mode and the system standing, the surface is zero and therefore its derivative and discontinuous part are zero. Hence, we deduce the expression of the equivalent control:

\[ u_{\text{eq}}(t) = -\left\{ \frac{\partial s}{\partial t} g(x,t) \right\}^{-1} \left\{ \frac{\partial s}{\partial x} f(x,t) \right\}. \tag{9} \]

For the equivalent control can take a finite value, must \( \frac{\partial s}{\partial x} g(x,t) \neq 0 \).

During the convergence mode and the replacing of the equivalent control by its expression in (8), we find the new expression of the derivative of the surface:

\[ \dot{s}(x) = \frac{\partial s}{\partial x} \left\{ g(x,t)\Delta u \right\}. \tag{10} \]

The problem is to find \( \Delta u \) such that:

\[ s(x)\dot{s}(x) = s(x) \frac{\partial s}{\partial x} \left\{ g(x,t)\Delta u \right\} < 0. \tag{11} \]

The simplest form that can take the discrete control is a relay. In this case the attractive control is as follows:

\[ \Delta u = -k \text{sign}(s(x,t)). \tag{12} \]
Substituting (12) into (11), we obtain:

\[ s(x)\dot{s}(x) = \frac{\partial s}{\partial t} g(x,t)k|s(x)| < 0. \]  \hspace{1cm} (13)

It is necessary that \( \frac{\partial s}{\partial t} g(x,t)k|s(x)| < 0 \) to satisfy the conditions of attractiveness of the sliding surface. The gain \( k \) is chosen to satisfy the positive condition (13). The choice to gain great influence because it is very small response time is very long, and if chosen very large, we will have large oscillations at the organ of the control. These oscillations can excite the dynamics disregarded (the chattering phenomenon) [3, 4, 16].

4 Chattering Phenomenon

In practice, an ideal sliding mode does not exist because the switching frequency of the control devices has a finite limit. In other words, there is not any switching device may switch to an infinite frequency (indeed this organ predicted to deliver an infinite energy) [19].

The discontinuous term \( U_n \) can excite unmodelled high frequency dynamics around a boundary layer of the sliding surface which involve the appearance of the chattering phenomenon, and which is characterized by large oscillations around the Surface (Fig. 1).

\[ \begin{array}{c}
\begin{array}{c}
\text{Chattering Phenomenon} \\
\text{(converges to the desired state)}
\end{array}
\end{array} \]

Fig. 1 – Chattering phenomenon.

From the perspective of the synthesis control, it is common to choose the switching surface \( s(x) = 0 \) by fixing most frequently the sliding dynamics and we deduce a discontinuous control \( U_n \) which makes the attractive surface and thus provides the appearance of the sliding mode. Despite the various advantages of the sliding mode control, its use has been hampered by a major disadvantage to the phenomenon of chattering. This is a natural consequence of the dynamic behavior of the real assembly system-actuator to be controlled.
The chattering phenomenon can result from deterioration anticipated of the control system or excite high frequency dynamics not considered in the modeling of system.

Thus, we search for different methods to limit this phenomenon. One approach is to replace the sign function by an intelligent control \[4, 6, 17, 19\].

### 5 Design of Fuzzy Sliding Mode Controllers

The conventional sliding mode control is based on the discontinuous function of state variables in the system that is used to create a “sliding surface”. When this surface is reached, the discontinuous function keeps the trajectory on the surface of such so that the desired system dynamics is obtained \[1, 14, 15\].

In this paper, the controllers of speed and rotor flux are substituted by a fuzzy sliding mode control to obtain a robust performance. By keeping one part of the equivalent control (SMC) and adding the fuzzy logic control (FLC) we obtain the new method control (FSMC) as shown in Fig. 2 \[17\].

\[
u_{FSMC} = u_{eq} + u_{fuzzy}.
\]

The two parts are combined to provide stability and robustness of the system, the method of control by fuzzy logic approach is adopted to solve the problem of Chattering.

![Fig. 2 – The structure diagram of the hybrid control FSMC.](image)

#### 5.1 Synthesis of the SMC controllers

In the design of sliding mode control of speed and rotor flux of the system, the switching function is chosen as follows \[3\]:

\[
\begin{align*}
  s_1 &= k_1 e_\omega + \dot{e}_\omega \\
  s_2 &= k_2 e_\phi + \dot{e}_\phi
\end{align*}
\]
with:

\[ e_\omega = \omega - \omega_{\text{ref}}, \quad e_{\phi_r} = \phi_r - \phi_{\text{ref}}. \]  

(16)

Beginning with the replacement of (16) into (15) we have:

\[ s_1 = k_1(\omega - \omega_{\text{ref}}) + (\dot{\omega} - \dot{\omega}_{\text{ref}}), \]

\[ s_2 = k_2(\phi_r - \phi_{\text{ref}}) + (2\dot{\phi}_{ra} + \dot{\phi}_{\text{ref}}) - \dot{\phi}_{\text{ref}}. \]

(17)

After substitution of (17) and (5) we arrive at:

\[ S_1 = \frac{k_1}{\mu} (\omega - \omega_{\text{ref}}) + (i_{sl} \phi_{ra} - i_{sa} \phi_{rb}) - \frac{T_L}{\mu} - \frac{\dot{\omega}_{\text{ref}}}{\mu}, \]

\[ S_2 = \frac{T_r}{2} k_2 (\phi_r - \phi_{\text{ref}}) + [M(i_{sa} \phi_{ra} + i_{sl} \phi_{rb}) - \phi_r] - \frac{T_r}{2} \dot{\phi}_{\text{ref}}, \]

where \( k_1 \) and \( k_2 \) are positive gains.

The development of calculated derivatives of the surfaces gives:

\[ \dot{s}_1 = \left(k_1 - \frac{1}{T_r} - \gamma\right) \mu f_2 - k_1 \frac{T_L}{J} - p\mu \omega (f_1 + K \phi_r) - \]

\[ -k_1 \dot{\omega}_{\text{ref}} - \ddot{\phi} + \alpha \mu \phi_{ra} \phi_{sa} - \alpha \mu \phi_{rb} V_{sb}, \]

\[ \dot{s}_2 = \frac{2}{T_r} \left( \frac{T_r k_2}{2} - 1 \right) \dot{\phi}_r + \frac{2M}{T_r} \left( \frac{M}{T_r} f_3 - \left( \frac{1}{T_r} + \gamma \right) f_1 + K \phi_r + p \omega f_2 \right) - \]

\[ -k_2 \dot{\phi}_{\text{ref}} - \ddot{\phi}_{\text{ref}} + \frac{2\alpha}{T_r} M \phi_{rb} V_{sb} + \frac{2\alpha}{T_r} M \phi_{ra} V_{sa}, \]

with:

\[ f_1 = i_{sa} \phi_{ra} + i_{sl} \phi_{rb}, \quad f_2 = i_{sl} \phi_{ra} + i_{sa} \phi_{rb}, \quad f_3 = i_{sa}^2 + i_{sl}^2. \]

(20)

The necessary condition for the states system follows the trajectory defined by the sliding surfaces is \( s_i = 0 \), the equivalent part \( u_{\text{eq}} \) is the control to providing \( \dot{s} = 0 \) for the nominal system \( \dot{s} = 0 \) give:

\[ \left(k_1 - \frac{1}{T_r} - \gamma\right) f_2 - k_1 \frac{T_L}{\mu J} - p\omega (f_1 + K \phi_r) - \frac{k_1}{\mu} \dot{\phi}_{\text{ref}} - \frac{1}{\mu} \ddot{\phi} = \]

\[ = \alpha \phi_{rb} V_{sa} - \alpha \phi_{ra} V_{sb}, \]

\[ \left( \frac{T_r k_2}{2} - 1 \right) \dot{\phi}_r + M \left( \frac{M}{T_r} f_3 - \left( \frac{1}{T_r} + \gamma \right) f_1 + K \phi_r + p \omega f_2 \right) - \]

\[ -\frac{T_r}{2} k_2 \dot{\phi}_{\text{ref}} - \frac{T_r}{2} \ddot{\phi}_{\text{ref}} = \alpha M \phi_{rb} V_{sb} + \alpha M \phi_{ra} V_{sa}. \]

(21)
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Assume:

\[
A = \left( k_1 - \frac{1}{T_r} - \gamma \right) f_2 - k_1 \frac{T_L}{\mu J} - p \omega (f_1 + K \varphi) - k_1 \frac{\dot{\omega}}{\mu} - \frac{1}{\mu} \ddot{\omega},
\]

\[
B = \left( \frac{T_r k_2}{2} - 1 \right) \phi_r + M \left( \frac{M}{T_r} f_3 - \left( \frac{1}{T_r} + \gamma \right) f_1 + \frac{K}{T_r} \varphi + p \omega f_2 \right) - \frac{T_r}{2} k_3 \dot{\phi} - \frac{T_r}{2} \ddot{\phi} \tag{22}
\]

or:

\[
\dot{s} = 0 \quad \Rightarrow \quad \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -\alpha \varphi_{r\beta} & \alpha \varphi_{r\alpha} \\ \alpha M \varphi_{r\alpha} & \alpha M \varphi_{r\beta} \end{bmatrix} \begin{bmatrix} V_{sa} \\ V_{s\beta} \end{bmatrix} \tag{23}
\]

We have

\[
F = \begin{bmatrix} A \\ B \end{bmatrix} \quad \text{and} \quad \Rightarrow \quad \begin{bmatrix} V_{sa} \\ V_{s\beta} \end{bmatrix} = -E^{-1} F = u_{eq} \tag{24}
\]

5.2 Design of FLC controllers

It is well known as one of the disadvantages of the SMC is the Chattering phenomenon. In this section, a fuzzy control FLC is introduced to replace the function \( K_{1,2} \cdot \text{sign}(s_{1,2}) \), as known the trajectory of state can reach and move along the surface of change, a good dynamic steady state can be achieved by the combination of SMC and FLC [11, 14].

The fuzzy controller used in this paper is two inputs and one output as shown in Fig. 3.

![Fig. 3 – Block diagram of fuzzy logic controller.](image)

The membership functions are defined in Fig. 4a and Fig. 4b. The fuzzy rule base consists of a collection of linguistic rules of the form [7, 11]:

Rule 1: if \( s_{1,2} \) is NB and \( ds_{1,2} \) is NB then \( dU_{1,2} \) is NB
Rule 2: if $s_{1,2}$ is NM and $ds_{1,2}$ is NB then $dU_{1,2}$ is NB
Rule 3: if $s_{1,2}$ is NS and $ds_{1,2}$ is NB then $dU_{1,2}$ is NS
Rule 49: if $s_{1,2}$ is PB and $ds_{1,2}$ is PB then $dU_{1,2}$ is PB

These inferences can be made in a more explicit table, called decision table [9].

Fig. 4 – Membership function:
(a) input; (b) output.
Table 1
Fuzzy Inference Table.

<table>
<thead>
<tr>
<th>$dU_{1,2}$</th>
<th>$ds_{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NB</td>
</tr>
<tr>
<td>NB</td>
<td>NB</td>
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<tr>
<td>NM</td>
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<td>NS</td>
<td>NB</td>
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<tr>
<td>ZE</td>
<td>NB</td>
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<tr>
<td>PS</td>
<td>NM</td>
</tr>
<tr>
<td>PM</td>
<td>NS</td>
</tr>
<tr>
<td>PB</td>
<td>ZE</td>
</tr>
</tbody>
</table>

6 Sensitivity Study and Simulation Results

To conclude the performance of the new method using the principle of a hybrid control by fuzzy sliding mode, we will present simulations of an induction motor controlled by a voltage inverter. This performance was established from the simulations of operating modes: Speed variation test with application of a load torque and robustness of the control relative to parametric variations (stator and rotor resistances) and variation of inertia follows by a variation of a load torque.

Fig. 5 – Block diagram of the proposed FSMC.
6.1. Speed variation test with application of a load torque

To show the robust fuzzy sliding mode performances FSMC we have simulated the system described in Fig. 5.

Fig. 6 – Simulation results under different speed references with application of a load torque.
The first test concerns the speed evolution and the disturbance rejection of FSMC and SMC controllers. This test is related to the performances of the drive system at low reference speed. When the induction motor is operated at 100 rad/sec under no load and a load torque 5 N.m is suddenly applied at t= 0.3 s, followed by a consign inversion -100 rad/sec at t= 0.4 s and accelerate again to 15 rad/sec. When the induction motor operates in low speed we eliminate the torque load in t= 0.8 s.

Fig. 6 shows clearly that the FSMC gives good performances, rejects the load disturbance very rapidly with no overshoot, with a negligible steady state error, maintain the decoupling between a torque-flux and reduce the chattering phenomenon.

### 6.2. Robustness test

In order to test the robustness of the used approach we have studied the effect of the parameters uncertainties on the performances and we compared with the SMC control.

To show the effect of the parameters uncertainties, we have simulated the system with different values of the parameter considered and compared to nominal value.

![Stator and rotor resistance variations](image_url)
Two cases are considered:

1. The stator and rotor resistances (50% and 100% respectively) as shown in Fig. 7.

2. The moment of inertia (50% to 100%).

To illustrate the performances of control, we have simulated the starting mode of the motor without load, and the application of the load \( T_L = 5 \text{ Nm} \) at \( t_1 = 0.3 \text{ s} \) and its elimination at \( t_2 = 0.7 \text{ s} \), in presence of the variation of parameters considered (the moment of inertia, the stator and rotor resistances) with speed step of 100 rad/s.

Fig. 8 shows the system response realized with the SMC and FSMC for different values of stator and rotor resistances; it is obviously that the FSMC for the regulation of speed is better than the conventional control SMC. We remark that the start-up speed, dynamics performance and the robustness of the new approach are all good Fig. 8a, the torque chattering in proposed approach FSMC and conventional control SMC during the variation of parameters and the application of the load are 3 Nm to 4 Nm and 5 Nm to 13 Nm respectively Fig. 8a and Fig. 8b. So the advantage of the proposed method control is obvious.

Fig. 9 shows the tests of robustness realized with the SMC and FSMC for different values of the moment of inertia. For the robustness of control, a variation of the moment of inertia \( J \), doesn’t have any effects on the performances of the used approach.

![Fig. 8 – Simulation results under stator and rotor resistances variation: (a) FSMC control; (b) SMC Control.](image)
7 Conclusion

The paper describes a new approach to robust control for induction motor. It develops a simple robust controller to deal with uncertain parameters and external disturbances. The control strategy is based on SMC and FLC approaches. The FSMC has the advantage in handling the torque-ripple phenomenon and in reducing the number of the fuzzy rules. The simulation results show that the proposed controller is superior to SMC in minimization of torque-ripple with varying parameters and with higher tracking precision. It appears from the response properties that it has a high performance in presence of the uncertain parameters plant and load disturbances. It is used to control system with unknown model. The FSMC control gives fast dynamic response with no overshoot and zero steady-state error. The decoupling between torque-flux is verified.

8 References


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