Authors acknowledge the useful comments of two anonymous referees.

Exchange Rate Misalignment, Capital Accumulation and Income Distribution
Theory and Evidence from the Case of Brazil

Summary: This article analyzes the relationship between economic growth, income distribution and real exchange rate within the neo-Kaleckian literature, through the construction of a nonlinear macrodynamic model for an open economy in which investment in fixed capital is assumed to be a quadratic function of the real exchange rate. The model demonstrates that the prevailing regime of accumulation in a given economy depends on the type of currency misalignment, so if the real exchange rate is overvalued, then the regime of accumulation will be profit-led, but if the exchange rate is undervalued, then the accumulation regime is wage-led. Subsequently, the adherence of the theoretical model to data is tested for Brazil in the period 1994/Q3-2008/Q4. The econometric results are consistent with the theoretical non-linear specification of the investment function used in the model, so that we can define the existence of a real exchange rate that maximizes the rate of capital accumulation for the Brazilian economy. From the estimate of this optimal rate we show that the real exchange rate is overvalued in 1994/Q3-2001/Q1 and 2005/Q4-2008/Q4 and undervalued in the period 2001/Q2-2005/Q3. As a direct corollary of this result, it follows that the prevailing regime of accumulation in the Brazilian economy after the last quarter of 2005 is profit-led.

Key words: Open economy macroeconomics, Income distribution and real exchange rate.


In recent years, some interesting literature has been developed about the relationship between real exchange rate (hereafter RER) and economic growth. For example, the seminal article by Ofair Razin and Susan Collins (1997) points to the existence of important non-linearities in the relationship between the variables of exchange rate misalignment – defined as a lasting deviation of the RER with respect to some reference value, determined by “fundamentals” – and real output growth from a sample of 93 developed and developing countries, in the period 1975-1993. The empirical results show that, while only very severe overvaluations of the RER are associated with slower economic growth in the long run, moderate undervaluations of the RER have a positive effect on the growth of the gross domestic product (GDP). Dani Rodrik (2008), when analyzing the development strategies adopted by a number of countries, noted that an important factor for initiating a process of sustained growth in real
output is to maintain a stable, depreciated RER. Similarly, Roberto Frenkel (2004),
when analyzing the employment performance and growth rates of Argentina, Brazil,
Chile and Mexico, concluded that maintaining a competitive and stable RER is the
best contribution macroeconomic policy can make to long-term economic growth.
Barry Eichengreen (2008) reached similar conclusions regarding the output in the
long run and the undervaluation of the RER.

In post-Keynesian literature, the relationship between the real exchange rate
and growth has been largely neglected. In the context of the so-called balance-of-
payments constrained growth model, originally developed by Anthony Philip Thirl-
wall (1979), the long-term equilibrium growth rate depends on the ratio between the
income elasticity of exports and imports multiplied by the growth rate of the income
of the rest of the world. Variations in the real exchange rate are assumed to be irrele-
vant for long-term growth, since empirical evidence shows either that price elastic-
ities of exports and imports are low, meaning that the impact of a real devaluation of
the exchange rate on the growth rate of exports and imports is small, or that terms of
trade do not show a systematic trend of appreciation or depreciation in the long run
(John Stuart Landreth McCombie and Mark Roberts 2002, p. 92).

In the context of the so-called neo-Kaleckian models of growth and income
distribution, the level of the RER can affect the long-term growth, due to the impact
of this variable on functional income distribution. Indeed, if a profit-led accumula-
tion regime prevails, a devaluation of the RER will result in both a higher degree of
productive capacity utilization and a higher investment rate. This result is due to the
fact that a devaluation of the RER will cause a reduction in real wages, thereby in-
creasing firms’ profit margins, which, in turn, has a positive effect on their planned
investments (Amit Bhaduri and Stephen Marglin 1990; Robert Blecker 2002). A fall
in real wages, however, will reduce consumer demand due to the fact that propensity
to consume out of wages is higher than propensity to consume out of profits. How-
ever, if the sensitivity of investment to variations in the profit margins is high and if
the difference between the propensities to consume out of wages and profits is small,
then the drop in consumer demand induced by the reduction in real wages will be
more than offset by increased investment demand, thereby causing an increase in the
degree of productive capacity utilization. Otherwise, the reduction in real wages pro-
duced by the devaluation of the RER will result in a decrease in the degree of pro-
ductive capacity utilization, which generates negative effects on investment and, con-
sequently, on the growth rate of real output due to the so-called “accelerator effect”.
In this case, the economy will be operating in a wage-led regime.

The neo-Kaleckian literature, however, disregards the existence of possible
non-linearities in the relationship between the RER and long-term growth, such that
the existence of different accumulation regimes over time in the same economy is
rejected a priori. In other words, it would not be possible for a capitalist economy to
move from a wage-led accumulation regime to a profit-led accumulation regime, and
vice versa, as a result of (exogenous) variations in the RER. Moreover, in the models
from this literature, the nature of the accumulation regime is independent not only
from the RER level, but also from the degree of misalignment with respect to some
reference value of the exchange rate under consideration.
This article aims to advance the analysis of the relationship between economic growth, income distribution, and the RER within the neo-Kaleckian literature, through the construction of a non-linear macro-dynamic model for an open economy in which investment in fixed capital is assumed to be a quadratic function of the RER. This particular shape of the investment function is intended to represent the effect that non-linear variations of the RER have on investment and, consequently, on long-term growth. In fact, for very low levels of the RER, devaluations of the RER have positive effects on investment, given there is an increase in the competitiveness of domestic products in international markets. For very high levels of the RER, however, devaluations of the RER have negative effects on investment since it increases the cost of imported machinery and equipment, discouraging investment – especially in developing countries which are heavily dependent on capital goods imports for the technological upgrading of their companies.

Based on this specification of investment function, it is demonstrated that the prevailing accumulation regime in a given economy depends on the type of exchange rate misalignment – defined as the difference between the actual value of the RER (the amount of national currency corresponding to a unit of foreign currency, i.e.: number of Reais/US$) and the value of that rate that maximizes the rate of capital accumulation – observed in the economy under consideration. Indeed, if the RER is overvalued, then the accumulation regime is profit-led, thus indicating that the reduction in real wages brought about by a devaluation of the RER will prompt an increase in both the degree of productive capacity utilization and the rate of capital accumulation. Inversely, if the RER is undervalued, then the accumulation regime is wage-led, such that the increase in real wages and in the share of wages in income resulting from an appreciation of the RER will prompt an increase in both the degree of productive capacity utilization and the rate of capital accumulation.

Subsequently, the adherence of the theoretical model to data is tested for the Brazilian case in the period 1994/Q3 to 2008/Q4. The econometric results confirm the existence of a statistically significant non-linearity between the rate of capital accumulation and the RER for the period under consideration. More precisely, the econometric results are consistent with the theoretical specification of the investment function used in the neo-Kaleckian model presented in this article; therefore, it is possible to determine the existence of an RER that maximizes the rate of capital accumulation in the Brazilian economy. From the estimation of this “optimal” rate, the RER is found to be overvalued in the periods 1994/Q3-2001/Q1 and 2005/Q4-2008/Q4, and undervalued in the period 2001/Q2-2005/Q3. As a direct corollary of this result, it follows that the prevailing accumulation regime in the Brazilian economy since the last quarter of 2005 has been profit-led such that the process of increasing the share of wages in income since 2004 has served to reduce, rather than stimulate, capital accumulation and long-term growth of the Brazilian economy.

The remainder of this article is organized into five sections. Section 1 presents the basic structure and the equilibrium solution of the neo-Kaleckian growth model and income distribution. Section 2 is dedicated to estimating the equation of capital accumulation for the Brazilian economy in the period 1995/Q3-2008/Q4, in which there exists a non-linear and statistically significant relationship between capital
stock growth rate and RER. Section 3 analyzes the “optimal” RER, that is, the RER that maximizes the rate of capital accumulation in the Brazilian economy. From the calculation of this optimal rate, we can identify the sub-periods in which the RER was undervalued and the sub-periods in which it was overvalued. Section 4 offers a reprise of the conclusions of this article.

1. Real Exchange Rate, Income Distribution and Capacity Utilization in a Neo-Kaleckian Open Economy Model

1.1 Price Formation and Income Distribution

We consider a small open economy that produces a homogeneous good, which serves for both consumption and investment, from labor and intermediate inputs imported from abroad. Firms are assumed to be price makers, such that the prices of their products are fixed based on a mark-up over direct unitary cost of production, as observed in equation (1) below:

\[ p = (1 + z)[w a_1 + e p^* a_0]. \]  

Where:
- \( p \) is the price of the domestic good,
- \( z \) is the rate of mark-up,
- \( w \) is the nominal wage rate,
- \( e \) is the nominal exchange rate,
- \( p^* \) is the price of imported input in the currency of the country of origin,
- \( a_0 \) is the unit requirement of imported inputs, and
- \( a_1 \) is the unit requirement of the workforce.

We will assume that the final good produced in the economy under consideration is an imperfect substitute of the final goods produced abroad, such that international trade does not impose the law of one price on tradable goods. In other words, the purchasing power parity does not apply. However, the monopoly power of domestic companies is affected by the price of imported goods. More specifically, the ability of domestic firms have to set a price above the direct unitary cost of production depends on the RER, which is defined as the ratio between the price of imported goods in the domestic currency and the price of the domestic goods in the domestic currency. In this context, a devaluation of the RER allows domestic companies to increase the mark-up on production costs due to the reduced competitiveness of the final goods imported from abroad.

Thus, we can express the mark-up as a function of the RER as follows:

\[ z = z_0 + z_1 \theta. \]  

Where: \( \theta = \frac{e p^*}{p} \) is the real exchange rate.

Dividing expression (1) by \( p \), we obtain:

\[ 1 = (1 + z)[V a_1 + \theta a_0]. \]  

Equation (3) shows the distributive locus of the economy under consideration, that is to say, the combinations of real wage \( (V) \), the real exchange rate \( (\theta) \) and the mark-up rate \( (z) \) for which the added value produced in the economy is entirely embodied in the form of wages and profits. Note that, given the mark-up and the techni-
cal coefficients of production, there is an inverse relationship between the real wage and the RER. In other words, a devaluation of the RER is accompanied necessarily by a reduction in real wages. As the mark-up rate depends positively on the RER, it follows that the required reduction in real wages will be even greater than in the case of the mark-up being fixed.

In fact, substituting (2) into (3) and differentiating the resulting expression with respect to V and $\theta$, we obtain:

$$\frac{\partial V}{\partial \theta} = -\left\{ \frac{a_0 (1 + z_0) + z_1 (a_1 V + 2a_0 \theta)}{a_1 (1 + z_0) + z_1 a_1 \theta} \right\} < 0. \quad (4)$$

The distribution of income between wages and profits also depends on the RER. Indeed, the share of profits in income is given by:

$$\pi = \frac{z}{1+z} = \frac{z_0 + z_1 \theta}{1 + z_0 + z_1 \theta}. \quad (5)$$

By differentiating (5) with respect to $\pi$ and $\theta$, we get:

$$\frac{\partial \pi}{\partial \theta} = \frac{z_1}{(1 + z_0 + z_1 \theta)^2} > 0. \quad (6)$$

In other words, a devaluation of the RER promotes an increase in the share of profits in income.

1.2 Effective Demand

Like Michael Kalecki (1954), Nicholas Kaldor (1956) and Joan Robinson (1962), we will assume there are two classes, capitalists and workers, which have different propensities to consume. Capitalists consume a constant fraction $c_p$ of their disposable income, composed only of profits, while workers consume a constant fraction $c_w$ of their disposable income, composed only of wages. By hypothesis, the capitalists’ propensity to consume is less than the workers’ propensity to consume. The government levies taxes on both wages and profits, but the tax rate on wages is supposedly to be less than the tax rate on profits. Thus, the nominal expenditure on consumption is given by:

$$pC = c_w (1 - \tau_w) wL + c_p (1 - \tau_p) \pi pX. \quad (7)$$

Where: $L$ is the quantity of labor employed, $\tau_w$ is the tax rate on wages, $\tau_p$ is the tax rate on profits, $X$ is the real income.

By dividing expression (7) with $pK$, we obtain after the necessary calculations:

$$\frac{C}{K} = \left\{ c_w (1 - \tau_w) - \pi [c_w (1 - \tau_w) - c_p (1 - \tau_p)] \right\} u. \quad (8)$$
Where:  

\[ K \] is the capital stock,  

\[ u = X/K \] is the degree of productive capacity utilization.

By differentiating (8) with respect to \( \pi \), we get:

\[
\frac{\partial (C/K)}{\partial \pi} = -[c_w(1 - \tau_w) - c_p(1 - \tau_p)]u < 0. 
\]  

Expression (9) shows that an increase in the share of profits in income – induced, for example, by a devaluation in the RER – will result in a contraction in consumption per unit of capital given that (i) the propensity of workers to consume is greater than the propensity of capitalists to consume, and (ii) the tax rate on wages is less than the tax rate on profits. It thus follows that, regarding consumer spending, a devaluation of the RER has a contractionary effect on effective demand.

As in Bhaduri and Marglin (1990), the desired rate of growth of capital stock is assumed to be a separable function of the share of profits in income and the degree of productive capacity utilization. However, the RER has an effect on the decision of capital accumulation that goes beyond the impact it has on the share of profits in income, such that its influence on the rate of capital accumulation will be modeled separately from the other two variables due to the non-linearity that the RER introduces in the investment decision. In fact, up to a certain point a depreciated RER stimulates investment because it allows for an increase in monopoly power of domestic firms and, therefore, an increase in mark-ups and the share of profits in income. The increase in profitability will induce a higher rate of capital accumulation. However, since part of the capital equipment necessary for the realization of investment spending is imported from abroad, then a too much depreciated exchange rate may discourage investment decisions due to the higher prices of imported capital goods. It thus follows that, from a certain critical level of the RER, investment becomes an inverse function of the RER since the effect of “the increased costs of imported capital equipment” tends to surpass the effect of “the increase in the profit margins of domestic firms”.

Based on this reasoning, we can write the desired rate of capital accumulation\(^1\) as follows:

\[
g = \frac{I}{K} = \alpha_0 + \alpha_1 \pi + \alpha_2 u + \alpha_3 \theta - \alpha_4 \theta^2. 
\]  

By differentiating (10) with respect to \( \theta \), we get:

---

\(^1\) In equation (10) above we are supposing that the growth rate of capital stock is independent of the composition of capital stock between domestic and imported capital goods. This assumption can be easily justified if we consider a Leontief technology for the production of aggregate capital. More precisely, we suppose that domestic and imported capital goods are used in fixed coefficients for the production of aggregate capital, that is  

\[ K = \min \{k_d, k_f\}, \]  

where \( k_d \) is the quantity of domestic capital goods used in the production of aggregate capital and \( k_f \) is the quantity of imported capital goods used in the production of aggregate capital. Since domestic and imported capital goods have used in fixed proportions, a RER depreciation will reduce the growth rate of aggregate capital stock since firms can not substitute imported for domestic capital in order to increase their productive capacity.
Expression (11) allows us to define the RER that maximizes the rate of capital accumulation in the economy under consideration, according to expression (12) below:

$$\theta^* = \frac{\alpha_3}{2\alpha_4}.$$  

From (12) it is possible to define the exchange rate overvaluation as a situation in which the RER is below $\theta^*$ and the exchange rate undervaluation as a situation in which the RER is above $\theta^*$.

Figure 1 below gives a visualization of the relationship between the rate of capital accumulation and the RER.

![Figure 1: Capital Accumulation and the RER](Image)

Source: Own elaboration.

With regard to government spending, we will assume that fiscal policy is guided by the generation of primary surplus targets in order to guarantee the stability and/or reduction of the public sector net debt as a proportion of the capital stock\(^2\). Thus, government expenditures are determined in a purely residual way, losing the “autonomy” that characterizes them in the Keynesian/Kaleckian inspired models.

In this context, $f$ is government expenditures (consumption and investment) as a proportion of the capital stock, $\tau$ tax collections as a proportion of the capital stock, and $d$ primary surplus targets as a proportion of the capital stock. We get:

$$\bar{d} = \tau - f$$  

$$\tau = \{\tau_w + (\tau_p - \tau_w)\pi\}u$$  

$$f = \{\tau_w + (\tau_p - \tau_w)\pi\}u - \bar{d}.$$  

---

\(^2\) This policy is necessary to maintain the inter-temporal solvency of the government budget.
In equation (15) it is observed that, given the primary surplus target as a proportion of the capital stock and the tax rates on profits and wages, government expenditures as a proportion of the capital stock are highly pro-cyclical since they tend to increase with the degree of productive capacity utilization\(^3\). We also observed in equation (15) that, given the degree of productive capacity utilization, government expenditures are an increasing function of the share of profits in income. It, thus, follows that, all else kept constant, a devaluation of the RER should produce an increase in government expenditures as a proportion of the capital stock, thereby contributing to the increase in effective demand.

Finally, we will assume that net exports as a proportion of the capital stock are given by the following expression:

\[
\frac{E}{K} = \varepsilon_0 + \varepsilon_1 \theta - \varepsilon_2 u. \tag{16}
\]

Where: \(\varepsilon_0, \varepsilon_1\) and \(\varepsilon_2\) are positive constants.

In equation (16) we are assuming that the Marshall-Lerner condition holds such that a devaluation of the RER produces an increase in net exports.

### 1.3 Equilibrium Solution of the Model

The goods market will be in equilibrium when the effective demand equals the production level of the companies. As such, we get:

\[
u = \frac{C}{K} + \frac{I}{K} + \frac{G}{K} + \frac{E}{K}. \tag{17}\]

Substituting (8), (10), (15) and (16) into (17), we get:

\[
u^* = \frac{\alpha_0 + \varepsilon_0 - \tilde{d} + \alpha_1\pi(\theta) + (\varepsilon_1 + \varepsilon_3)\theta - \alpha_4\theta^2}{1 - c_w(1 - \tau_w) + \pi(\theta)[c_w(1 - \tau_w) - c_p(1 - \tau_c) - (\tau_c - \tau_w)] - \tau_w - \alpha_1 + \varepsilon_2}. \tag{18}\]

### 1.4 Effects of a Variation in the RER

By differentiating expression (18) with respect to \(u\) and \(\theta\), we get:

\[
\frac{\partial u^*}{\partial \theta} = \frac{\alpha_1 \pi’ + (\varepsilon_1 + \varepsilon_3) - 2\alpha_4 \theta - \pi \left[c_w - c_p \right]c_w(1 - \tau_w) - \tau_w(1 - c_w) + \tau_p(1 - c_p) - \varepsilon_2}{\left[1 - c_w(1 - \tau_w) + \pi(\theta)[c_w - c_p] + \tau_w(1 - c_w) - \tau_p(1 - c_p) - \tau_w - \alpha_1 + \varepsilon_2\right]^2}. \tag{19}\]

We observed in expression (19) that the effect of an exchange rate devaluation on the short-term equilibrium of the degree of productive capacity utilization depends on the level of the RER. In particular, we will show below that the sign of the partial derivative depends on the relationship between the RER and the value that maximizes the desired rate of capital accumulation.

\(^3\) This is because we are assuming that \(t_c > \tau_w\).
In order for the partial derivative in (19) to have a positive sign, it is necessary that the following condition to be met:

\[
\theta < \left( \frac{\alpha_3}{2\alpha_4} \right) + \frac{\left( \alpha_1 \pi' + \varepsilon_1 \right)}{2\alpha_4} - \pi \left[ (c_w - c_p) + \tau_p (1 - c_p) - \tau_w (1 - c_w) \right] u^* = \theta^{**}.
\] (20)

That is:

\[
\theta < \theta^* + \zeta = \theta^{**}.
\] (21)

While the constant term in (21) is positive\(^4\), it follows that the critical value of the RER, below which the partial derivative in (19) is positive (\(\theta^{**}\)), is greater than the critical value of the RER that maximizes the desired rate of capital accumulation (\(\theta^*\)). From here it follows that, in a situation in which the RER is overvalued, that is, \(\theta < \theta^*\), a devaluation of the RER will increase not only the desired rate of capital accumulation but also the degree of productive capacity utilization.

With this reasoning, it follows that, in the case where the exchange rate is overvalued, the accumulation regime is profit-led, since a devaluation of the RER will result in an increase in the share of profits in income (Eq. 6), as well as an increase in the degree of productive capacity utilization and the rate of capital accumulation. In the case where the exchange rate is undervalued, an appreciation of the RER will result in an increase in the degree of productive capacity utilization (and the rate of capital accumulation) and a reduction in the share of profits in income. In this case, the accumulation regime is wage-led.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Accumulation Regimes and Exchange Rate Misalignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate misalignment</td>
<td>Accumulation regime</td>
</tr>
<tr>
<td>Overvalued exchange rate</td>
<td>Profit-led</td>
</tr>
<tr>
<td>Undervalued exchange rate</td>
<td>Wage-led</td>
</tr>
</tbody>
</table>

\(^4\) A sufficient condition for this result is that: \(\left[ (c_w - c_p) + \tau_p (1 - c_p) - \tau_w (1 - c_w) \right] < 0\).

2. Econometric Modeling

The theoretical model (Equation 10) will be translated into the following econometric specification:

\[
g = I / K = \alpha_0 + \alpha_1 \pi + \alpha_2 u + \alpha_3 \theta + \alpha_4 \theta^2 + \varepsilon.
\]

Where: \(g\) is the rate of accumulation, which corresponds to the ratio of gross fixed capital \((I)\) to the stock of fixed productive capital \((K)\) available to the Brazilian economy; \(\pi\) is the profit share and corresponds to the share of profits in the GDP; \(u\) is the level of installed productive capacity utilization, corresponding to the deseasonalized time series produced by the National Confederation of Industry (CNI); \(\theta\) is the RER between the Brazilian real and the American dollar; and \(\theta^2\) is the square of the previous time series. The description of the variables used in the econometric model is found in Table 2.
Table 2  Description of the Variables and Data Source

<table>
<thead>
<tr>
<th>Notation</th>
<th>Comments</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock ($K$)</td>
<td>Stock of productive fixed capital. Corresponds to the sum of the stock in machinery and equipment plus the stock in non-residential buildings.</td>
<td>Institute of Applied Economic Research (IPEADATA)</td>
</tr>
<tr>
<td>Profit share ($\pi$)</td>
<td>Corresponds to the portion of the profits ($L$) in the Gross Domestic Product (GDP).</td>
<td>Brazilian Institute of Geography and Statistics (IBGE)</td>
</tr>
<tr>
<td>Rate of productive fixed capital accumulation ($g$)</td>
<td>Corresponds to the ratio of gross fixed capital formation and the stock of productive fixed capital ($K$) available in the Brazilian economy.</td>
<td>IPEADATA and IBGE</td>
</tr>
<tr>
<td>Level of utilization of installed productive capacity ($u$)</td>
<td>Corresponds to the deseasonalized time series produced by the National Confederation of Industry (CNI).</td>
<td>IPEADATA</td>
</tr>
<tr>
<td>The real exchange rate ($\theta$)</td>
<td>The RER is the product between the nominal exchange rate of Brazil in relation to the American dollar and the ratio of USA consumer price index to Brazilian consumer price index. Real exchange rate squared ($\theta^2$): RER squared $\theta \times \theta$.</td>
<td>International Financial Statistics of International Monetary Fund</td>
</tr>
<tr>
<td>Total profits ($L$)</td>
<td>Corresponds to the non-agricultural GDP minus wages ($W$).</td>
<td>IBGE</td>
</tr>
<tr>
<td>Investment ($I$)</td>
<td>Gross fixed capital formation/GDP.</td>
<td>IPEADATA</td>
</tr>
</tbody>
</table>

Source: Own elaboration.

Following the theoretical model, the following signs are expected:

$$\frac{\partial g}{\partial \pi} > 0; \frac{\partial g}{\partial u} > 0; \frac{\partial g}{\partial \theta} > 0; \frac{\partial^2 g}{\partial \theta^2} < 0.$$

Before estimating the theoretical model, the variables must be tested for their order of integration. The test employed for this purpose was the Augmented Dickey-Fuller test proposed by Said E. Said and David Dickey (1984). The results are shown in Table 3.

Table 3  Unit Root Test, ADF

<table>
<thead>
<tr>
<th>Variables</th>
<th>lags</th>
<th>t test</th>
<th>Critical value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>log0</td>
<td>0</td>
<td>-1.57</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>log02</td>
<td>0</td>
<td>-1.34</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>Logh</td>
<td>3</td>
<td>-1.06</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>Log</td>
<td>1</td>
<td>-2.46</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>Logu</td>
<td>0</td>
<td>-2.23</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>lags</th>
<th>t test</th>
<th>Critical value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>dlog0</td>
<td>0</td>
<td>-6.39</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>dlog02</td>
<td>1</td>
<td>-6.04</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>dlogh</td>
<td>1</td>
<td>-3.69</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>dlogg</td>
<td>0</td>
<td>-5.88</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
<tr>
<td>dlogu</td>
<td>0</td>
<td>-6.27</td>
<td>-3.55</td>
<td>-2.91</td>
</tr>
</tbody>
</table>

Source: Own elaboration – research data.
The null hypothesis \( (H_0) \), where the time series tested have unit root (is not stationary), is not rejected for all variables (in level) with statistical significance of 1%. Therefore, it is possible to differentiate the time series as many times as necessary and perform the unit root test to know when the series become stationary, finding, thus, their order of integration. The table also shows the results of the ADF test for the first difference of the time series. The calculated \( t \) statistics allow us to reject the null hypothesis, indicating that the time series are stationary in the first difference and, therefore, integrated of order one, or \( I(1) \).

After verifying, by means of the unit root test, that the time series are not stationary and that they have the same order of integration, it is possible to perform the cointegration test, in order to verify if a linear combination of these variables is stationary (Søren Johansen 1988; Johansen and Katarina Juselius 1990). Table 4 shows the results of the cointegration test.

### Table 4: Johansen Test

<table>
<thead>
<tr>
<th></th>
<th>Trace statistics</th>
<th>Maximum eigenvalue statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Critical value 5%</td>
</tr>
<tr>
<td>( R=0 )</td>
<td>90.96719</td>
<td>88.80380</td>
</tr>
<tr>
<td>( R \leq 1 )</td>
<td>49.06038</td>
<td>63.87610</td>
</tr>
<tr>
<td>( R \leq 2 )</td>
<td>24.98199</td>
<td>42.91525</td>
</tr>
</tbody>
</table>

Source: Own elaboration – research data.

The null hypothesis (there is no cointegrating relationship) is rejected at the 5% level of significance, both by the trace statistics and the maximum eigenvalue statistics, as shown in Table 3. In other words, there is strong evidence in favor of the existence of a cointegrating vector, which represents the long-term relationship between the variables in question. Given the evidence of cointegration between the variables, and that they are \( I(1) \), the next item is to estimate an OLS model and a VEC model with the time series in level\(^5\). The results are shown in Table 5.

### Table 5: Estimated Model for Brazil – Rate of Capital Accumulation as Dependent Variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>MQO Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>Constant</td>
<td>-11.321***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([-6.13])</td>
</tr>
<tr>
<td>( Inh )</td>
<td>Log of the profit share</td>
<td>0.658*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([1.821])</td>
</tr>
<tr>
<td>( Inu )</td>
<td>Log of the level of capacity utilization</td>
<td>2.121***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([5.125])</td>
</tr>
<tr>
<td>( Ine )</td>
<td>Log of the real exchange rate</td>
<td>0.238***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([3.415])</td>
</tr>
<tr>
<td>( Lne2 )</td>
<td>Log of the real exchange rate squared</td>
<td>-0.123*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>([-4.144])</td>
</tr>
</tbody>
</table>

**Note 1:** MQO: R-squared: 0.8; Adjusted R-squared: 0.79; Durbin-Watson: 1.79; F-statistics: 61.22; Prob (F-statistics): 0.0000; number of observations: 57 after adjustments.

**Note 2:** *** Significant at 1 percent level; ** Significant at 5 percent level; * Significant at 10 percent level.

**Note 3:** All variables are lagged.

Source: Own elaboration – research data.

\(^5\) According to James Hamilton (1994), if the time series model has these characteristics, the OLS method continues to be a super-consistent estimator. For a formal demonstration in this respect, see Hamilton (1994, p. 587).
The result of the estimation shows that the empirical model is relatively well-adjusted. In general terms, all signs of the coefficients estimated are in accordance with the expected and in agreement with what was discussed in the theoretical section. The level of installed capacity utilization variable not only showed the expected sign but was also the variable with the largest coefficient among the ones included in the investment equation. The profit share variable had a positive sign, indicating that increases in the share of profits in income increases the rate of accumulation, thus defining the presence of a profit-led accumulation regime in the Brazilian economy in the period under consideration.

Regarding the RER and RER squared variables, it is observed that a devaluation of the RER implies an increase in the rate of capital accumulation as shown by the positive sign of $e$. However, the negative sign of the RER squared variable shows that this relationship is not linear, such that very high levels of the RER, instead of stimulating the rate of accumulation, leads to its reduction.

3. The Optimal Exchange Rate for the Brazilian Economy

In order to find the optimal real exchange rate, we have also to estimate the profit-share and the level of capacity utilization functions for Brazilian economy. We will do it by using the method of ordinary least squares. As proposed in the theoretical model, the profit-share is a linear function of the exchange rate and the level of capacity utilization is a quadratic function of the real exchange rate, which can be represented by the following econometric specification:

$$ h = \alpha_h + \beta_h \theta + h_i $$

$$ u = \alpha_u + \beta_u \theta + \gamma_u \theta^2 + u_i. $$

The results can be seen in Table 6.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.0135</td>
<td>0.008924</td>
<td>3.340481</td>
<td>0.0014</td>
</tr>
<tr>
<td>$C$</td>
<td>-0.5554</td>
<td>0.017769</td>
<td>29.03761</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.0416</td>
<td>2.729040</td>
<td>3.839254</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\theta^2$</td>
<td>-0.0479</td>
<td>0.567115</td>
<td>-3.645217</td>
<td>0.0005</td>
</tr>
<tr>
<td>$C$</td>
<td>4.4052</td>
<td>3.119288</td>
<td>21.78271</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Source: Own elaboration – research data.

Now we have three estimated equations for the log of rate of capital accumulation, profit share and capacity utilization, all dependent on the log of real exchange rate and/or the square of log real exchange rate.
Then we have:

\[
\log g = -11.3221 + 0.6825 \log h + 2.121 \log u + 0.238 \log \theta - 0.123 \log \theta^2 + g_i \tag{24}
\]

\[
\log h = -0.555404 + 0.01305 \log \theta + h_i \tag{25}
\]

\[
\log u = 4.405281 + 0.041625 \log \theta - 0.04792 \log \theta^2 + u_i \tag{26}
\]

Putting (25) and (26) in (24) we get:

\[
\log g = -11.321 + 0.685 (-0.0555 + 0.0013 \log \theta) + 2.121 (4.40 + 0.40 \log \theta - 0.048 \log \theta^2) + 0.238 \log \theta - 0.123 \log \theta^2 \tag{27}
\]

After some mathematical manipulations we get:

\[
\log g = -2.37 + 0.32 \log \theta - 0.224 \log \theta^2 \tag{28}
\]

Taking the derivative of (5) in respect to \( \log \theta \) and equaling to zero we get:

\[
\frac{\partial \log g}{\partial \log \theta} = 0.32 - 0.448 \log \theta = 0 \tag{29}
\]

\[
\log \theta = \frac{0.32}{0.448} = 0.71 \tag{30}
\]

So we conclude that optimal real exchange rate for Brazilian economy is given by: \( \theta = 2.03 \).

From the time series of the RER for the Brazilian economy, we can determine the moments in which the RER was undervalued or overvalued in the period 1994/Q3–2008/Q4. A visualization of the periods of exchange rate misalignment can be seen in Figure 2 below.

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6 For more information on the Brazilian exchange rate policy see André Moreira Cunha, Daniela Magalhães Prates, and Fernando Ferrari-Filho (2011).
As we can see in Figure 2 above, the RER was overvalued in the period 1994/Q3–2001/Q1 and 2004/Q4–2008/Q4 and undervalued in the period 2001/Q2–2004/Q3. We can also observe that from the third quarter of 2002 the RER starts a strong trend of appreciation which, based on the neo-Kaleckian growth and distribution model presented in the previous section, should lead to an acceleration in the rate of capital accumulation. In fact, this is precisely what happened, as can be seen in the graphs below that show the behavior of the macroeconomic time series used in the previous section. Indeed, we found that the growth rate of the capital stock shows a significant increase from early 2003 to late 2007, with a slight decrease thereafter. It thus follows that, in the sub-period in which the RER was undervalued, the accumulation regime was wage-led, since the simultaneous occurrence of an increase in the share of wages in income and an acceleration in the rate of capital accumulation is observed in this sub-period.

Note: U level of capacity utilization, G rate of accumulation and RER exchange rate.

**Figure 3** Evolution (in log) of the RER, the Rate of Capital Accumulation, the Degree of Capacity Utilization and the Share of Profits in Income in the Brazilian Economy (1994–2008)

Source: Own elaboration – research data.
4. Final Remarks

Throughout this article, we present a neo-Kaleckian model of growth and income distribution for an open economy in which the rate of capital accumulation is assumed to be a non-linear function (quadratic) of the RER. In this context, we demonstrated the existence of an optimal RER, that is, the RER that maximizes the rate of capital accumulation. In addition, it was argued that the accumulation regime will be wage-led when the RER is undervalued, in other words, above the value that maximizes the rate of capital accumulation; and profit-led when the RER is undervalued, in other words, below the “optimal” value for the rate under consideration.

Subsequently, we estimated the investment function used in the theoretical model with data from the Brazilian economy for the period 1994/Q3–2008/Q4. The econometric tests corroborated with the expected signs of the coefficients in the theoretical equation. In particular, there is a non-linear relationship statistically significant between the rate of capital accumulation and the RER. Based on the estimated coefficients of the capital accumulation function, we calculated the “optimal” RER for the Brazilian economy, that is, the value of the RER that maximizes the rate of capital accumulation. From the estimated value of the “optimal” RER, we identified two sub-periods of exchange rate overvaluation (1994/Q3–2001/Q1 and 2005/Q4–2008/Q4) and a period of exchange rate undervaluation (2001/Q2–2005/Q3). In the period of exchange rate undervaluation, the accumulation regime was wage-led since the simultaneous occurrence of an acceleration in the rate of capital accumulation and an increase in the share of wages in income was observed. The rate of capital accumulation started to reduce, however, at the end of 2007, in the second sub-period where the RER was overvalued. This may indicate, therefore, the exhaustion of the wage-led model and the emergence of a profit-led model of growth and capital accumulation.
References


