The objective of this article is to contribute to the theoretical analysis of the relationship between the level of real exchange rate and long-term growth in developing economies by means of a growth model that combines elements from both the Keynesian/Kaleckian and the Latin American Structuralist approaches. The main hypothesis that underlies the construction of our growth model is that maintaining a competitive real exchange rate induces investment and structural change in the economy, at the same time that allows relaxing the external constraint to long term growth given by balance of payments equilibrium condition. This means that exchange rate policy can influence growth not only through an increase of short-term competitiveness, but also by providing the necessary incentives for investment and technological development. This implies that exchange rate policy is capable of influencing the long-term supply-side conditions, as it is capable of inducing a change in income elasticities of exports and imports.
The article is organized in six sections. In Section 1 we will present a brief review of empirical and theoretical (heterodox) literature about the relationship between the level of real exchange rate and economic growth. In Section 2 we will develop a model of capital accumulation and income distribution in order to analyse the effects of changes in the level of real exchange rate over the pace of capital accumulation. In Section 3 we make a brief review of the literature that explores the nexus between structural change, capital accumulation and real exchange rate. In Section 4 we present a balance of payments constrained growth model with endogenous income elasticities of exports and imports. Section 5 combines the models developed in Sections 2 and 4 in a single Keynesian-Structuralist growth model in order to analyse the effects of different exchange rate policies over the pace of capital accumulation and economic growth. Section 6 made a brief review of the results obtained through out the paper.

1. Real Exchange Rate and Economic Growth: A Review of the Literature

We have recently been seeing the growth of an important literature on the relationship between the real exchange rate and growth. Ofair Razin and Susan M. Collins (1999) indicated that there are important nonlinearities in the relationship between exchange rate misalignments - defined as long-lasting deviations of the real exchange rate from an underlying reference value, given by “fundamentals” - and the real growth of output. Their study employed a sample of 93 developed and developing countries, for the 1975-1993 period. The empirical results show that very high overvaluations are associated to lower growth in the long-term, whereas, on the other hand, moderate under-valuations positively impact GPD growth. Dani Rodrik (2008) based in a panel of 188 developed and developing countries from 1950 to 2004 showed that real exchange rate undervaluation had positive and statistically significant effects over growth rate of per capita income of these countries, even after controlling for variables as domestic saving rate, terms of trade and government consumption as a percent of GDP. He also shows that the effect of undervaluation is strongest for developing than developed countries. Paulo Gala (2008) finds a negative relationship between growth of GDP per capita and a PPP-based index of real exchange rate overvaluation in a panel of 58 developing countries for the period 1960-1999. His findings were robust to changes in control variables and econometric techniques. Martin Rapetti, Peter Skott, and Arslan Razmi (2012) showed that Rodrik’s findings are sensitive to the criterion used do divide the sample between developed and developing countries. Based on alternative classification criteria and empirical strategies to evaluate the existence of asymmetries between groups of countries, they find that the effect of exchange rate undervaluation on growth is indeed larger and more robust for developing countries than were found in Rodrik’s paper. Sophie Béreau, Antonia López Villavicencio, and Valérie Mignon (2012) showed, through a Panel Smooth Transition Regression Model, that real exchange rate misalignments have differentiated impact on economic growth depending on their sign: whereas overvaluation negatively affect economic growth, real exchange
rate undervaluation significantly increases it. Regarding the Brazilian case, José Luis Oreiro, Lionello F. Punzo, and Eliane Araujo (2012) found that exchange rate misalignments had a negative and statistically significant effect on the growth rate of real output for the 1994-2007 period.

The relationship between the real exchange rate and growth is, however, being neglected in the post-Keynesian growth literature. In the so-called balance of payments-constrained growth models, which Anthony P. Thirlwall (1979) pioneered, the long-term equilibrium growth rate depends on the ratio between the income elasticities of exports and imports. In these type of models that are no mechanism by which the level of real exchange rate can affect long-term growth; only the rate of change of real exchange rate can do it. But these is also considered irrelevant to long-term growth either because empirical work had shown that price elasticities of exports and imports are low, hence a positive rate of change of real exchange rate (a cumulative real exchange rate depreciation) would have nothing but a reduced impact on the growth rate of exports and imports; or terms of trade and real exchange rate do not display an upward or downward trend in the long-term, which means that long term growth rate of exports and imports do not depend on the rate of change of the real exchange rate but only on growth rate of foreign and domestic output (John S. L. McCombie and Mark Roberts 2002, p. 92).

Regarding the so-called neo-Kaleckian models of growth and income distribution, the level, instead the rate of change, of the real exchange rate can affect long-term growth, since the level of real exchange rate had a direct impact over income distribution. If a profit-led regime of accumulation prevails, than a real exchange rate devaluation will result in an increase of capacity utilisation and investment rate. This is due to the fact that devaluation of real exchange rate will reduce real wages and increases profit margin of firms, inducing an increase in their planned investment (Amit Bhaduri and Stephen Marglin 1990; Robert A. Blecker 2002). Lower wages will, for sure, reduce consumption demand, since workers propensity to consume is assumed to be higher than capitalists propensity to consume; but, if the difference between both propensities is small and investment is highly sensible to changes in the profit margin, then the fall in consumption demand due to lower wages will be more than offset by increased investment demand. This leads to an increase of capacity utilisation. Otherwise, lower real wages due to exchange rate devaluation will decrease capacity utilisation and investment demand. In this case, the economy can be said to operate in a wage-led regime.

Another way the real exchange rate can influence long-term growth, which is particularly important for developing economies, is through its impact on the degree of structural heterogeneity of these economies. Structural heterogeneity, as defined by Latin American Structuralist School of Thought, is a situation where an economy had only a small dynamic core of economic activities, restricted to relatively modern primary exports sector with a few associated manufacturing and service segments. The rest of the economy is characterized by a primitive occupational structure and high unemployment rate. These economies are at the same time specialised and heterogeneous. This is because structural heterogeneity refers to the technological and productivity differences inside the productive structure, which are largely the result...
of dynamic insufficiency of the system, caused by the slow pace of capital accumulation, by the adoption of inadequate technologies and by the wide variation of the quality of the workforce (see Octavio Rodriguez 2009).

It should be highlighted that, in this setting, the level of real exchange rate influences both capital accumulation and technological innovation, thereby establishing a connection between real exchange rate and growth from the supply-side of the economy. In fact, technology is the keystone of long-term growth, as improved production techniques lead to higher productivity and faster growth rates, which in turn allow for incorporating surplus labour and reducing structural heterogeneity. Structural change is, however, the effect of capital accumulation itself, since the latter reduces the technological gap (for the concept of technological gap, see Jan Fagerberg 1994) - given that, as a rule, new technologies are embodied in new machinery and equipment (Nickolas Kaldor 1957). The level of real exchange rate can induce technological and structural change by means of a higher investment rate. Since an increase in the level of real exchange rate - i.e. a real exchange rate depreciation - will induce an increase in the profit share, then it will increase internal funds and the self-financing capacity of firms, producing a reduction in borrowers and lenders risk and, thereby, stimulating a higher rate of capital accumulation.

2. A Model of Capital Accumulation, Income Distribution and Real Exchange Rate

We will consider a small developing economy that produces a single homogeneous good \([X]\), used for both consumption and investment. The inputs are labour \([N]\) and an imported raw material \([M]\). Firms in this economy are price-makers in goods market, fixing the price for a unit of homogeneous output by means of a mark-up over direct unitary costs of production. The price setting rule is show in Equation (1):

\[
p = (1 + z)[a_0w + a_1ep^*].
\]

(1)

Where: \(p\) is the price of the domestic good, \(z\) is the mark-up rate, \(w\) is the nominal wage rate, \(e\) is the nominal exchange rate, \(p^*\) is the price of the imported raw materials in foreign currency, \(a_0 = \frac{N}{X}\) is the unitary labour requirement and \(a_1 = \frac{M}{X}\) is the unitary requirement of raw material.

Let us define \(Y\) as the gross value of output in real terms and \(pY\) as the gross value of output in nominal terms. So we have the accounting identity given below:

\[
pX = pY - ep^*M.
\]

(2)

This means that \(pX\) is the net added value in nominal terms, and \(X\) is also the net added value in real terms.

Let us define \(\nu = \frac{w}{p}\) as the real wage rate and \(\theta = \frac{ep^*}{p}\) as the level of real exchange rate. The profit share is given by:

\[
h = \frac{pY - ep^*M - wN}{pX} = \frac{pX - wN}{pX} = 1 - \nu a_0.
\]

(3)
Dividing both sides of Equation (1) by $p$, we get:

$$1 = (1 + z)[(1 - h) + a_1 \theta].$$  \hspace{1cm} (4)

Solving Equation (4) for $h$, we get:

$$h = \left(\frac{z}{1+z}\right) + a_1 \theta.$$  \hspace{1cm} (5)

In Equation (5) we see that a devaluation of real exchange rate will increase profit share for a given mark-up rate.

Like Michael Kalecki (1971), Kaldor (1955-56) and Luigi L. Pasinetti (1962) we will suppose the existence of two social classes, workers and capitalists. Workers supply labour and receive wages as income which is fully spent in consumption. Capitalists earn only profits and save a constant share of them. Aggregate real savings [$S$] are thus defined as a fixed portion $S$ of capitalist profits [$P$], as shown in Equation (6).

$$S = sP = s \frac{P}{X} X^P K.$$  \hspace{1cm} (6)

Where: $X^P$ is the level of real output that is compatible with full capacity utilization and $K$ is the capital stock of the economy.

Defining $u = \frac{X}{X^P}$ as the level of capacity utilization, $h = \frac{P}{X}$ as the profit share and $q = \frac{X^P K}{K}$ as the productivity of capital, we get:

$$\sigma = \frac{S}{K} = shuq.$$  \hspace{1cm} (7)

In Equation (7) $\sigma$ is aggregate saving as a ratio of capital stock. Without loss of generality we can set $q = 1$, so we get:

$$\sigma = \frac{S}{K} = shu.$$  \hspace{1cm} (7a)

As we can see in Equation (7a) since the aggregate saving rate is a positive function of the profit share, a devaluation of real exchange rate will induce an increase in the saving rate of the economy as a whole. This occurs because the propensity to save out of profits is higher than the propensity to save out of wages, so an income redistribution from wages to profits - due to the exchange rate devaluation - will cause a reduction in aggregate consumption and, hence, an increase in the saving rate.

Regarding investment behaviour, we will suppose that the growth rate of capital stock that is desired by capitalists is given by:

$$g = \gamma + \alpha_1 u + \alpha_2 h + \alpha_3 \theta - \alpha_4 \theta^2.$$  \hspace{1cm} (8)

Where: $g$ is the desired growth rate of capital stock, $\gamma$ represents “autonomous” part of investment, determined by “animal spirits”.

The specification of investment equation follows Bhaduri and Marglin (1990) on taking the desired rate of capital stock as a separable function of profit share and capacity utilization; contrary to the standard procedure used in Kaleckian growth

Our innovation here consists in introducing the level of the real exchange rate as an independent argument of the investment function. Furthermore we also suppose that the square of real exchange rate, not only its level, affects investment behaviour. This means that growth rate of capital stock is a non-linear function of the level of real exchange rate. The non-linearity is based on the idea that, on one hand, currency devaluations positively affect the competitiveness and profitability of tradable sectors, thus stimulating firms that produce exportable goods to invest in capacity expansion and in the acquisition of new production techniques. The argument here is that technological progress should be considered, to a great extent, endogenous to variations of the level of the real exchange rate. The technological gap can be reduced by acquiring foreign technology or by developing new processes and innovations internal to the firm, in both cases levered by the greater availability of funds (profitability). Nevertheless, we also consider that technological progress can also occur through capital accumulation, for new technologies are, as a rule, embodied in new machinery and equipment. On the other hand, currency devaluation also increases the costs of imported inputs, including machinery and equipment, thereby increasing the cost of investment and reducing the desired growth rate of capital stock. There is no reason to believe that these opposite effects cancel each other. It is more reasonable to think that for very low levels of real exchange rate, the competitiveness and profitability of tradable sectors are also very low, discouraging investment in new machines and equipment, as a result the growth rate of capital stock is also low. For very high levels of real exchange rate, however, the cost of investment will be very high due to high prices of imported machines and equipment. As a result, the growth rate of capital stock will again be low. In this case, for intermediate levels of real exchange rate competitiveness, profitability and the cost of investment will be at reasonable levels in order to induce a high rate of capital accumulation. In order to formalize this non-linear effect of real exchange rate over capital accumulation, the growth rate of capital stock is supposed to be a square function of real exchange rate.

Gilberto Tadeu Lima and Gabriel Porcile (2013) had also developed a dynamic model of growth and capacity utilisation that takes into account the joint determination of international competitiveness (measured by the real exchange rate) and the functional distribution of income. As regards our current model, this means that the accumulation function (the investment function) should not be specified with $h$ and $\theta$ as independent terms. In what follows we will consider the case where $\alpha_2 = 0$, that is we will exclude profit share from the accumulation equation.

Following Oreiro and Araujo (2013), we will suppose that net exports as a ratio of capital stock $[\varphi]$ are given by:

$$\varphi = \varphi_0 + \varphi_1 \theta - \varphi_2 u.$$  \hspace{1cm} (9)

Where: $\varphi_0, \varphi_1, \varphi_2 > 0$.

In Equation (9) we are assuming that the Marshall-Lerner condition holds such that a devaluation of the real exchange rate an increase in net exports.
Regarding inflation, we will consider that the rate of change of domestic prices is equal to the rate of change of nominal wages minus the rate of change of labor productivity:

$$\hat{\rho} = \hat{w} - \hat{\delta}. \tag{10}$$

Where: $\hat{\rho}$ is the rate of inflation, $\hat{w}$ is the nominal wage inflation and $\hat{\delta}$ is the growth rate of labor productivity.

Following Dutt (1994) we will suppose that over time the money wage changes according to the gap between the wage share targeted by workers, $w^*, s$, and the actual wage share and the expected rate of inflation. Since $(ws = 1 - h)$, we can write the following Equation for the rate of change of nominal wages:

$$\hat{w} = \mu_1 (h - h^*) + \mu_2 \hat{p}^e. \tag{11}$$

Where: $\mu_1 > 0$ and $\mu_2 < 1$.

Substituting (11) in (10) and assuming perfect foresight ($\hat{p}^e = \hat{\rho}$) as Dutt (1994) we get:

$$\hat{\rho} = \frac{\mu_1}{1-\mu_2} (h - h^*) - \frac{\hat{\delta}}{1-\mu_2}. \tag{12}$$

In Equation (12) we can see that the equilibrium rate of inflation is a function of the gap between the actual level of profit share and the profit share that is targeted by workers. Since profit share is a positive function of the level of real exchange rate, we can conclude that a devaluation of real exchange rate is followed by a permanent increase in the rate of inflation. This result is due to real wage resistance, that is, the attempt of workers to preserve their real wages (and wage share in income) by means of bid up money wages as a reaction to offset the effect of currency devaluation over real wages (Mark Setterfield 1997, p. 62). The real wage resistance will be higher (and the increase in the rate of inflation) as higher is the magnitude of the coefficient $\mu_1$.

Considering an open economy without government activities, the short-run equilibrium condition is given by the equality between planned savings and investment, that is:

$$g + \phi = \sigma. \tag{13}$$

After substituting (7a), (8) and (9) in (13) we get the short-run equilibrium value for capacity utilization, considering the simplest case where $\alpha_2 = 0$:

$$u = \frac{(y + \phi_0) + (\alpha_3 + \phi_1) \theta - \alpha_4 \theta^2}{sh - \alpha_4 - \phi_2}. \tag{14}$$

In order for the short-run equilibrium to be stable is necessary to assume that: $sh - \alpha_1 - \phi_2 > 0$, that is propensity to save out of profits must be higher than propensity to invest out of profits (Skott 2010, p. 110).

Equation (10) represents short-term equilibrium value of capacity utilisation, indicating the level of capacity utilization that makes planned investment equal to the
savings of capitalists. In other words, it is the IS curve for an equilibrium trade balance without the government.

In Equation (5) we can define \( \frac{z}{1+z} = \beta_0 \) and \( a_1 = \beta_1 \). So we get:

\[
h = \beta_0 + \beta_1 \theta. \tag{5a}
\]

After substituting (5a) in (14), we get:

\[
u = \frac{(y + \varphi_0) + (\alpha_3 + \varphi_1)\theta - \alpha_4 \theta^2}{s(\beta_0 + \beta_1 \theta) - \alpha_1 - \varphi_2}. \tag{15}
\]

In order to get the short-run equilibrium value for the growth rate of capital stock is necessary to put (15) in (8). Then we get:

\[
g = \frac{g_0 + g_1 \theta + g_2 \theta^2 - g_3 \theta^3}{\Delta}. \tag{16}
\]

Where: \( g_0 = s\beta_0(y + \varphi_0) > 0; \) \( g_1 = s(\beta_1 y + (\alpha_3 + \varphi_1)\beta_0) > 0; \) \( g_2 = s((\alpha_3 + \varphi_1)\beta_1 - \beta_0 \alpha_4) = ?; \) \( g_4 = s\beta_1 \alpha_4 > 0; \) \( \Delta = s(\beta_0 + \beta_1 \theta) - \alpha_1 > 0. \)

In order to analyse the relation between the short-run equilibrium value of the growth rate of capital stock and the level of real exchange rate let us do a numerical simulation of the model, imposing the following value for the parameters of the model (Table 1).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Values Used in the Numerical Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>( s )</td>
<td>0.7</td>
</tr>
<tr>
<td>( y )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0.4</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.075</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>0.005</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>0.02</td>
</tr>
<tr>
<td>( \varphi_0 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \varphi_1 )</td>
<td>0.005</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Source: Authors' own elaboration.

One important remark about the values presented above. The relatively high value for the propensity to save out of profits is based on the estimates of Kaldor (1966, p. 312).

For these numerical values, the relationship between the growth rate of capital stock and the level of real exchange rate if given by a hump shaped curve as we can see in Figure 1 below.

In Figure 1 we can see that there is a level of real exchange rate that maximizes the growth rate of capital stock. Let \( \Theta^* \) this optimal level of real exchange rate. If real exchange rate is over-valued, that is if is below \( \Theta^* \), then the growth rate of capital stock can be increased by means of a devaluation of real exchange rate. On the
other hand, if real exchange rate is under-valued, that is if it is above $\theta^*$, than the rate of capital accumulation can be increased by means of an appreciation of real exchange rate.

![Figure 1 Growth Rate of Capital Stock as a Function of Real Exchange Rate](image)

**Figure 1** Growth Rate of Capital Stock as a Function of Real Exchange Rate

### 3. Productive Heterogeneity, Balance of Payments Constraint and Structural Change

We admit throughout this article that a devaluation of real exchange rate affects the economy’s productive heterogeneity and, consequently, the income elasticities of exports and imports. The hypothesis that both elasticities are endogenous has been recently taken up in the literature (Thomas I. Palley 2002; Nelson H. Barbosa-Filho 2006; Alberto Botta 2009; Ricardo A. Araujo 2012; Marcos A. R. Ferrari, Fábio N. P. Freitas, and Barbosa-Filho 2013; Luiz Carlos Bresser-Pereira, Oreiro, and Nelson Marconi 2014). We consider, nevertheless, that they are endogenous to the level of the real exchange rate, a hypothesis that Fabricio Missio and Frederico G. Jayme Jr. (2012) and Bresser-Pereira, Oreiro, and Marconi (2015) had developed.

According to Bresser-Pereira, Oreiro, and Marconi (2014, 2015) a devaluation of real exchange rate affects the productive heterogeneity of the economy as it reduces the relative real wages and the unit labour costs. This will change the level of international specialization, increasing the number of goods that are manufactured in the country and, consequently, the share of manufacturing industry in GDP. As a consequence of that income elasticity of exports will increase and income elasticity of imports will decrease, thereby increasing the growth rate of real output that is compatible with the balance of payments equilibrium.

Missio and Jayme Jr. (2012) admit that the income elasticity of the demand for exports is a direct function of, amongst others, the number of goods produced by a country and the level of technology embodied in them. In line with the preceding
work, they consider that variations of the real exchange rate affect real wages, which leads to a diversification or specialisation of the economy. This means that when real wages rise, for example, the sectors already in a disadvantaged position in the international market, given the low technological content of their goods, lose certain markets or cease to exist altogether. This forces the economy to specialise in sectors with natural comparative advantages. For developing economies, this means specialising in natural primary goods. Since income elasticity of the demand for exports of these goods is low; then specialising in natural primary goods will heightens the balance of payments constraint to growth. On the other hand, reducing real wages (a devaluation of real exchange rate) leads to a productive diversification, which in the long-term implies greater export capacity and lower dependency on imports.

The authors also highlight the fact that maintaining a competitive real exchange rate may strongly induce technological progress. More specifically, they argue that a devaluation of real exchange rate, as it increases the profits and self-financing capacity of firms, increases the funds available for investment projects related to research and development. In other terms, the argument goes that an overvalued currency is associated to a (temporary) redistribution of income in favour of wages, which implies that firms will have lower self-financing capacity. This, in turn, reduces their funds for acquiring new technologies and their access to external finance, since information asymmetries in financial markets generate credit rationing. Thus, even in face of the possibility of acquiring inexpensive technology abroad, it is likely that various sectors will be unable to invest in modernising their productive structures, in light of the lack of self-financing capacity and credit rationing. On that account, it is with a competitive currency that one expects firms to undergo innovative activities leading to greater productive heterogeneity (a greater scope of produced goods, for example) and also to structural homogenisation, for technological progress is then incorporated in sectors dissociated from the world market. Since the return of innovative activities is higher in more backwards sectors, the discontinuities are expected to be rapidly overcome.

They also defend that structural change can be brought about by capital accumulation itself. The latter reduces the technological gap, since new technologies are, as a rule, embodied in new machinery and equipment. Capital accumulation in turn critically depends on macroeconomic policies, especially an exchange rate policy focused on preserving the competitiveness of domestic industries. To demonstrate this argument the authors developed a model with endogenous elasticities of the demand for exports and imports, which depend on the average age of the economy’s capital stock. It is assumed that the newer or more modern is the capital stock the greater will be the technological content of the goods produced and, therefore, the higher will be the income elasticity for the demand for exports and the lower will be the income elasticity of the demand for imports. This means that a capital accumulation effort, with an impact on the productive structure via the modernisation of its manufacturing base, will increase the technological content of exports and, hence, will also raise the income elasticity of the demand for exports and the growth rate compatible with balance of payments equilibrium.
Lastly, it should be noted that increasing the presence of tradable sectors as a consequence of maintaining a competitive real exchange rate will increase the effects of dynamic economies of scale associated with the so-called Kaldor-Verdoorn Law. According to this law, there is a positive relationship between the growth of manufacturing output and the growth rate of productivity in manufacturing, with causality running from the former to the latter. In short, this happens because when output grows it brings about, through time, relevant transformations in the productive structure and in the composition of demand. This benefits manufacturing, for such transformations lead to using new production processes or developing new goods. Moreover, new firms appear and/or the existing ones grow, which enables them to use more modern equipment, possibly better suited to larger productive units.

The main point of this approach is that a demand-induced increase of output leads to productivity gains in sectors that display, in macroeconomic terms, dynamic economies of scale. We highlight that these economies of scale are associated to technological changes, and they are not, therefore, reversible. They mostly arise due to learning by doing and to the growing division of labour market growth brings about. It is thus the case that maintaining a competitive real exchange rate, insofar as it raises foreign demand, leads to a faster growth of output and productivity. This revisits the idea that there is a cumulative causation based on the mutual feedback between growth and increasing returns, associated to the greater technological progress the expansion of output induces. The growth of manufacturing sectors that maintaining a competitive exchange rate brings about would stimulate productivity gains and contribute to accelerate the rate of technological change of all the economy, increasing its competitiveness in the foreign market. Additionally, the increase of productive heterogeneity in a “dual” economy à la Lewis allows for increasing labour productivity by relocating workers from non-tradable, backwards sectors to advanced, tradable sectors.

Therefore, to sum up, we argue that maintaining a competitive real exchange rate increases productive heterogeneity, technological progress, self-financing capacity of investment and labour productivity. In the long-term, this leads to a higher income elasticity of the demand for exports. An analogous argument can be made for the income elasticity of the demand for imports, which is an inverse function of the number of goods the country produces and of the technology they embody. Consequently, a devaluation of real exchange rate, as it increases the productive heterogeneity and the technological content embodied in the goods, reduces the necessity of importing foreign goods, decreasing the income elasticity of the demand for imports.

4. A Balance of Payments Constrained Growth Model with Endogenous Elasticities

Following the literature on balance of payments-constrained growth (see Thirlwall 2002, Chapters 4-5), the demand for exports and imports are given by the following equations:
\[ X = \left( \frac{P_d}{EP_f} \right)^\eta (Y_f)^\varepsilon, \quad (17) \]

\[ M = \left( \frac{\varepsilon P_f}{P_{d, t}} \right)^\psi Y^\pi, \quad (18) \]

where: \( P_d \) is the price of domestic output, \( X \) is the quantum of exports, \( P_f \) is the price of foreign output, \( E \) is the nominal exchange rate, \( M \) is the quantum of imports, \( Y \) is domestic real output, \( Y_f \) is the foreign real output, \( \psi \) \((<0)\) is the price elasticity of the demand for imports, \( \pi \) is the income elasticity of the demand for imports, \( \eta \) \((<0)\) is the price elasticity of the demand for exports and \( \varepsilon \) is the income elasticity of the demand for exports.

Assuming zero capital mobility, current account equilibrium is given by:

\[ P_d X = P_f M E. \quad (19) \]

Taking the rate of change of Equations (13), (14) and (15) we get:

\[ g_x = \eta (\hat{P}_d - \hat{e} - \hat{P}_f) + \varepsilon g_{y,f}, \quad (20) \]

\[ g_m = \psi (\hat{e} + \hat{P}_f - \hat{P}_d) + \pi g_y, \quad (21) \]

\[ \hat{P}_d + g_x = \hat{e} + \hat{P}_f + g_m. \quad (22) \]

Where: \( g_x \) is the rate of growth of exports, \( g_m \) is the growth rate of imports, \( g_{y,f} \) is the growth rate of the rest of the world, \( g_y \) is the growth rate of domestic output, \( \hat{e} \) is the rate of change of nominal exchange rate, \( \hat{P}_d \) is the rate of change of domestic price and \( \hat{P}_f \) is the rate of change of foreign price.

Assuming that relative prices measured in a common currency remains unchanged in the long-run (Thirlwall 2002, p. 71) than we can set: \((\hat{e} + \hat{P}_f - \hat{P}_d) = 0\). Putting (20) and (21) in (22) we get:

\[ g_y = \frac{\varepsilon}{\pi} g_{y,f}. \quad (23) \]

Equation (23) states that the growth rate of real output that is compatible with balance of payments equilibrium in the long-run is given by the ratio of income elasticity of exports and income elasticity of imports multiplied by the growth rate of the rest of the world. This is the so-called Thirlwall’s law.

The difference with Thirlwall’s original work is that we will consider, based on the discussion made in last section, the case where income elasticities exports and imports are endogenous to the level of the real exchange rate as in Equations (24) and (25) bellow:

\[ \varepsilon = \varepsilon(\theta); \quad \frac{\partial \varepsilon}{\partial \theta} > 0, \quad (24) \]

\[ \pi = \pi(\theta); \quad \frac{\partial \pi}{\partial \theta} < 0. \quad (25) \]

Substituting (24) and (25) in (23) we get:
\[ g_y = \frac{\varepsilon(\theta)}{\pi(\theta)} g_{y,f}. \]  

(26)

It can be easily shown that \( \frac{\partial g_y}{\partial \theta} > 0 \), that is an increase in the level of real exchange rate (a devaluation of real exchange rate) will increase the growth rate of output that is compatible with the balance of payments equilibrium in the long-run.

The relation between balance of payments equilibrium growth rate and real exchange rate [BP curve] is shown in Figure 2.

![Figure 2 Balance of Payments Equilibrium Growth Rate as a Function of Real Exchange Rate](source)

5. A Keynesian-Structuralist Growth Model and Exchange Rate Policy

We will now combine the models developed in Sections 2 and 4 in a single Keynesian-Structuralist growth model. The model has two fundamental equations. The first one - Equation (12) - regards to the short-run equilibrium condition in goods market. This equation defines the growth rate of capital stock that is required for the equality between the growth rate of aggregate demand and growth rate of capital stock in order to produce a constant level of capacity utilization. The second one - Equation (22) - regards to the long-run equilibrium in the balance of payments. This equation defines the growth rate of real output that is compatible with balance of payments equilibrium.

In steady-state, output and capital stock must be growing at the same rate. This means that:

\[ g = g_y. \]  

(27)

And, in consequence, we get:

\[ g = \frac{g_0 + g_1 \theta + g_2 \theta^2 - g_3 \theta^3}{\Delta}, \]  

(16)

\[ g = \frac{\varepsilon(\theta)}{\pi(\theta)} g_{y,f}. \]  

(26)
Since the relation between capacity growth and real exchange rate is hump-shaped [IS curve], we will have two long-run equilibrium positions for the economy as we can see in Figure 3 below:

![Figure 3](image)

**Figure 3** Long-Run Equilibrium Values for Growth Rate and Real Exchange Rate

In Figure 3 $\theta_o$ is long-run equilibrium level of real exchange rate that corresponds to an over-valued currency - that is, a level of real exchange rate that is lower than the optimal level [$\theta^*$] - and $\theta_u$ is the long-run equilibrium level of real exchange rate that corresponds to an under-valued currency - that is, a level of real exchange rate that is higher than the optimal level of real exchange rate. As we can easily see in Figure 3 an equilibrium with under-valued real exchange rate is associated with a higher growth rate of real output compared to an equilibrium with over-valued real exchange rate. Under-valuation of real exchange rate is good for long-term growth.

As we have done in Section 2, we will now run a numerical simulation of the model. In order to do so, let us assume that income elasticities of exports and imports are given by:

$$\varepsilon(\theta) = \varepsilon_0 + \varepsilon_1 \theta, \quad (24a)$$

$$\pi(\theta) = \pi_0 - \pi_1 \theta. \quad (25a)$$

Table 2 shows the numerical values for the remaining parameters of the model:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_0$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>1.20</td>
</tr>
<tr>
<td>$\pi_1$</td>
<td>-0.01</td>
</tr>
<tr>
<td>$g_y,f$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Source:** Authors' own elaboration.
For the parameter values show in Tables 1 and 2, the long-run equilibrium positions of the economy can be visualized in Figure 4 below:

![Figure 4 Growth Rate of Capital Stock and Balance of Payments Equilibrium Growth Rate as a Function of Real Exchange Rate](image)

Source: Authors’ own elaboration.

Figure 4 Growth Rate of Capital Stock and Balance of Payments Equilibrium Growth Rate as a Function of Real Exchange Rate

In order to analyse the stability of long-run equilibrium we will suppose that the economy is always in short-run equilibrium, so that Equation (12) is continuously fulfilled. This means that our economy is always on the IS curve on Figure 3.

However the same is not true for our BP curve. In the short-run, the economy can run current account surpluses or deficits, the last ones being financed by loss of international reserves.

We will suppose that exchange rate regime is a *crawling peg*, in which monetary authorities set the rate of change of nominal exchange rate according to the following equation:

\[
\hat{e} = \hat{\theta}_T + (\hat{p}_d - \hat{p}_f).
\]

(28)

Where: \(\hat{\theta}_T\) is the desired rate of change of real exchange rate by monetary authorities.

The desired rate of change of real exchange rate is, for now, supposed to be a function of the difference between the long-run growth rate of exports and imports as we can see in Equation (24):

\[
\hat{\theta}_T = \delta(\pi(\theta)g - \varepsilon(\theta)g_{y,f}); \quad \delta < 0.
\]

(29)

Equation (24) states that monetary authorities desire to increase (decrease) real exchange rate when imports are growing at a faster (lower) rate than exports. In other words, monetary authorities are just reacting to balance of payments disequilibrium by means of adjusting the level of real exchange rate; as a matter of fact they are just copying the behavior of exchange rate in a floating exchange rate regime. Under this
“quasi-market rule”, a devaluation of real exchange rate will be executed by monetary authorities when:

\[ \hat{\theta}_T > 0 \Leftrightarrow g < \frac{e(\theta)}{\pi(\theta)} g_y. \] (30)

In other words, real exchange rate will be devalued when the growth rate of real output is lower than the balance of payments equilibrium growth rate; otherwise, real exchange rate will be appreciated.

With this dynamics for real exchange rate it can be easily demonstrated that the equilibrium with over-valued exchange rate is dynamic stable, and the equilibrium with under-valued exchange rate is dynamic unstable. This means that for levels of real exchange rate in the interval \((0, \theta_u)\), this economy shows a long-run tendency for exchange rate over-valuations, what seems to be a fundamental feature of medium-income economies (see Bresser-Pereira 2010). As a consequence of this tendency for over-valuation of real exchange rate, this economy will also have a lower growth rate than the one it could get with the same parameters or “fundamentals”.

This result, however, can be reversed if monetary authorities, instead of trying to replicate market behavior, set exchange rate policy in order to target some “desired” level for real exchange rate. In this case, the target could be precisely the level of real exchange rate in the under-valued long-run equilibrium. This means replacing Equation (29) by:

\[ \hat{\theta}_T = \delta(\theta - \theta_u); \delta < 0. \] (29a)

It is clear that under this exchange rate rule, the equilibrium with under-valued currency is now stable and the equilibrium with over-valued currency is unstable. This result shows that the exchange rate policy that is adequate for a robust economic growth in the long-run is to target real exchange rate at a competitive level, as suggested by Roberto Frenkel (2002).

Regarding the inflationary consequences of this exchange rate rule, we know from Equation (12) that a devaluation of real exchange rate is followed, due to real wage resistance, by a permanent increase in the level of inflation. This may be problematic for policy authorities mainly in countries where there is a formal or informal commitment to a certain level of inflation, in other words, in countries that had an explicit or implicit inflation targeting regime. This means that the implementation of such exchange rate rule demands the adoption of some kind of income policies, where a short-term decrease in real wages is negotiated with workers in trade for a higher level of employment and wage rate growth in the medium and long term. As shown by Bresser-Pereira, Oreiro, and Marconi (2015, Chapter 16) a short-term decrease in real wages may be of workers interest if it was followed by an increase in the rate of capital accumulation and productivity growth. In this case, the once-and-for-all decrease in the level of real wages due to devaluation of real exchange rate will be followed by a higher growth rate of real wages in the medium term. This means that in a relatively short period of time (six or seven years) the level of real wages will be higher than it would be if real exchange rate was not devalued.
6. Final Remarks

The present article developed a Keynesian-Structuralist growth model in order to analyse the long-run relationship between the level of real exchange rate and economic growth. The model combined some important features of the post Keynesian growth and distribution models as, for instance, the relation between pricing decisions, income distribution and capital accumulation; with some features of Latin American Structuralism like the emphasis on the relation between productive structure, external constraint and economic growth. Both theoretical traditions could be combined in the same growth model by one linking element: the idea that a faster economic growth requires structural change that can only be realized by means of a faster pace for capital accumulation. In this setting the level of real exchange rate can induce both a higher rate of capital accumulation and a change in the productive structure of the economy by means of increasing the number of goods that are produced inside domestic borders.

For plausible parameters values, it was show that the model had two long-run equilibrium positions, one with an over-valued currency and a low rate of economic growth; and another with an under-valued currency and a high rate of economic growth. If exchange rate policy is designed in such way that real exchange rate just reacts to balance of payments disequilibrium, than the over-valued equilibrium will be stable and the economy will show a long-run tendency for over-valuation of real exchange rate. However if exchange rate policy had a clear target for real exchange rate, than under-valued equilibrium will be stable and the economy will show a high rate of capital accumulation and economic growth. This means that the best contribution that macroeconomic policy can do for economic growth is to deliver a stable and competitive level for real exchange rate.
References


