Summary: Using frequency domain analysis, this paper examines the volatility spillover from the United States and Japanese stock markets to the Vietnamese stock market. Daily data of S&P 500, Nikkei 225 and VN-Index from January 01, 2012 to May 31, 2016 is used. In terms of estimation, the GARCH model is used to estimate volatilities in these stock markets; the Granger Causality Test is used to examine volatility spillover; and the test for causality in the frequency domain by Jorg Breitung and Bertrand Candelon (2006) is used to examine the volatility spillover at different frequencies. The empirical results provide two main contributions: (i) there is a significant volatility spillover from the United States to the Vietnamese stock markets, but the evidence of volatility spillover from the Japanese to the Vietnamese stock market is not found; and (ii) the volatility spillover may vary across frequency spectrum bands. To our best understanding, volatility spillover analysis using frequency domain approach was not previously reported in literature.

Keywords: causality, frequency domain, spillover, volatility.

JEL: C58, G15.
In this era of globalization, there may be a link between financial systems of countries. Volatility spillover is a transmission of volatility among stock markets. Therefore, volatility spillover tests are necessary for both investors and policy makers to have suitable information in their decision.

Although volatility spillovers from developed markets to other developed markets or emerging markets were confirmed in many studies such as Ng (2000), Miyakoshi (2003), Kharchenko and Tzvetkov (2013), Mohammadi and Tan (2015)…; to the extent of our knowledge, this effect was not found in previous literature. Vietnamese stock market, a frontier market operating in a transition economy and integrating more deeply into the world economy, could be different with other emerging markets. Therefore, the spillover from other markets to the Vietnamese stock market is worth exploring. The results will contribute additional knowledge about the integration of frontier markets in the transition economies into the global economy.

In reality, short- and long-term investors may have different considerations. Short-term investors focus more on the relationship at higher frequencies, whereas long-term investors look at the relationship at lower frequencies (Nikola Gradojevic 2013). Hence, long-term and short-term volatility spillovers should be analyzed separately to have more precise information for each investor. The frequency domain analysis, i.e., spectral analysis, can be used in this situation.

Despite its usefulness, frequency domain research, especially the volatility spillover analysis using frequency domain approach, is relatively scarce in the empirical finance literature (Gradojevic 2012). Aside from examining the volatility spillover from the United States and Japanese stock markets to the Vietnamese stock market (in time domain), this paper extends these test results by using the spectral analysis approach that helps short-term and long-term investors gather more precise information for their investment decisions.

1. Literature Review
   1.1. Volatility Spillover

Volatility is the measurement of the dispersion in a probability density (Carol Alexander, 2001). Therefore, the stock return volatility, which is the standard deviation of daily stock returns around the mean, measures the random variability of the stock returns (Choo, 2011). Accordingly, the higher the volatility, the higher the stock risk (Le Dinh Nghi, 2012). Thus, volatility plays an important role in managing portfolio risks. The GARCH model is widely used to estimate the volatility of stock returns.

Spillover effect is the result or effect of return and volatility of a market that have spread to another market (Choo, 2011). Hence, the volatility spillover is a transmission of volatility among stock markets. In other words, volatility spillover is a result of the interdependence among market economies. This interdependence means
that shocks, whether of global or local nature, can be transmitted across countries because of their financial linkages (Ahmed S. Abou-Zaid, 2011). The Granger Causality Test is a useful instrument to examine the volatility spillover between different financial markets (Zhou Pu, Lu Fengbin, and Wang Shouyang, 2014).

The volatility spillovers were tested in many previous studies. Angela Ng (2000) examined the volatility spillovers from Japan (proxy for regional markets) and the United States (proxy for world markets) to six Pacific–Basin equity markets such as Hong Kong, Korea, Malaysia, Singapore, Taiwan, and Thailand. The authors indicated that both regional and world factors are critical for market volatility in the Pacific–Basin region, although the world market influence tends to be greater. Similarly, Tatsuyoshi Miyakoshi (2003) also tested the volatility spillovers from Japan and the United States to seven Asian equity markets such as Korea, Taiwan, Singapore, Thailand, Indonesia, Malaysia, and Hong Kong and found that Japan and the United States transmitted the volatility to Asian markets. However, in contrast to the results found by Ng (2000), the volatility of the Asian markets is influenced more by the Japanese market than by the United States, and there was an adverse influence of the volatility from the Asian markets to the Japanese market. According to the authors, this may be due to a strong economic relationship between Japan and other Asian countries through the large amount of portfolio investment during the latter half of the 1990s. After that, Kharchenko & Tzvetkov (2013) tested the volatility spillover between developed (proxy by France, Germany and the United States) and emerging market (proxy by China, India and Russia) economies. The authors found some evidence that the volatility spillover moves in a unidirectional way from the developed to the emerging markets. A more recent finding was reported by Yusaku Nishimura, Yoshiro Tsutsui, & Kenjiro Hirayama (2015) indicating that China has a large impact on Japanese stocks via China-related firms in Japan. Hassan Mohammadi and Yuting Tan (2015) investigated the daily volatility spillover across equity markets in Mainland China (Shanghai and Shenzhen), Hong Kong and the United States. Using data from January 02, 2001 to February 8, 2013, the authors showed the unidirectional volatility spillover from the United States to the other three markets.

In summary, although the volatility spillovers across stock market indices have been studied over the last two decades, this topic is even more relevant now due to its practical importance and the nature of the volatility itself which varies over time (Larisa Yarovaya, Janusz Brzeszczyński, and Marco Lau 2016). Moreover, because the volatility spillover test from the United States and Japanese to the Vietnamese stock market was not found in previous studies, it is significant to examine this subject in this paper.

1.2. Frequency domain approach
Frequency is the number of occurrences of a repeating event per unit time. Theoretically, a time series can be composed of many time series with different frequencies. In reality, short- and long-term investors may have different considerations. Short-term investors focus more on the relationship at higher frequencies, i.e., short-term fluctuations, whereas long-term investors look at the relationship at lower frequencies, i.e., long-term fluctuations (Gradojevic, 2013). Therefore, techniques that can analyze long term and short term separately are needed. Frequency domain analysis, a technique that can analyze series at different frequencies, can be useful in this situation.

This method is based on the Fourier transform that transfers data from time domain to frequency domain and vice versa. In spite of being widely applied in natural and technical sciences such as physics, telecommunications, signal processing, etc., frequency domain analysis is relatively and scarcely used in empirical finance literature (Gradojevic, 2013). This approach is useful in causality analysis. Granger & Lin (1995) confirmed that the causality could be different at each frequency. Since the traditional Granger Causality Test could not explore these relations, the causality test in frequency domain is needed. The frequency domain causality test proposed by Breitung and Candelon (2006), which is based on the framework of John Geweke (1982) and Yozo Hosoya (1991), has been widely used recently. However, the authors indicated in this paper that their test had low power when the non-causality hypothesis was tested at a frequency close to 0 or pi. Fortunately, Hiroshi Yamada and Wei Yanfeng (2014) proved that the test of Breitung and Candelon (2006) is still useful even at such frequencies.

Using frequency domain approaches, some studies were developed to test the causal relations between economics time series. Yanfeng (2013) investigated the relationships between oil prices and the Japanese economy from a frequency domain perspective. The analysis showed that oil prices have nonlinear linkages with the Japanese economy at certain frequencies. The results suggested that oil prices have significant predictive power over industrial production, consumer price index and unemployment rates at low frequencies. Moreover, oil prices can predict industrial production and unemployment rates at some higher frequencies. This paper suggested that policy makers should pay more attention to the long-term effects of oil price shocks on Japan's economy. Gradojevic (2013) used the test for causality in the frequency domain by Breitung and Candelon (2006) to analyze the causal relationship between the returns on the stock market indices at Serbia, Croatia, Slovenia, Hungary and Germany. The results implied that there are relationships between stock markets, but not at all frequencies. For example, this study showed that the null hypothesis of no causality from the German market index returns to the Serbian market index returns...
is rejected for $\omega \in [0.6, 2.1]$, which represents the cycles between 3 and 11 days. Christophe Croux and Peter Reusens (2013) investigated whether stock prices can forecast the future domestic economic activity using Granger causality analysis in the frequency domain. Using 1991Q1–2010Q2 quarterly data, for the G-7 countries, the authors found that the low frequency components of the stock prices could contain information to forecast future GDP, while this is not the case with high frequency components. Jose Eduardo Gomez-Gonzalez et al. (2015) investigated the relationship between financial and real business cycles for 33 countries in the frequency domain. Applying the Granger-type causality tests in the frequency domain to data from both developed and emerging market economies, this paper characterized the relationship of credit and output cycles in the frequency domain. The authors indicated that the interdependence is highest at medium- and long-term frequencies. Moreover, in contrast to most of the previous literature, this paper explored the bidirectional Granger causality between them. Mustafa Ozer & Melik Kamisli (2016) examined the dynamic linkages between the Turkish financial markets by using the frequency domain causality analysis proposed by Breitung and Candelon (2006) for the weekly Turkish data from 2003 to 2015. The results show that volatility spillovers among stock market returns, interest rate, and Euro exchange rate vary at different frequencies. Jamal Bouoiyour and Refk Selmi (2016) investigated the strength and extent of causal relationship between BRICS (Brazil, Russia, India, China and South Africa) stock returns and real oil price using frequency domain approach. Using 1998-2015 quarterly data, the authors indicated that the impact of oil price on stock returns is not the same in BRICS countries. In more details, the results decomposed BRICS into three main groups: short-run causality from Oil to Stock Return (India and South Africa), medium- and long-run causality (China) and long-term relation (Brazil and Russia). Elie Bouri et al. (2017) used Breitung and Candelon (2006)’s approach to investigate short- and long-run causality across the implied volatility of crude oil and agricultural commodities. Using daily data of crude oil volatility index, the corn volatility index, and the wheat volatility index from July 27, 2012 to September 30, 2016, this paper also showed that the volatility causal relation differs between high and low frequencies.

In summary, because the causal relations can vary across the frequency bands (Granger and Lin 1995), the frequency domain analysis is needed to have a deeper insight into relationship between financial time series. Although volatility spillovers were reported in many previous studies, most of them could not analyze these effects at different cycles. To our best understanding, stock market volatility spillover analysis using frequency domain approach was not previously reported in the literature. Therefore, this paper aims to test the volatility spillover from the United States and
Japanese to Vietnamese stocks market, using a causality method in frequency domain proposed by Breitung & Candelon (2006). The results will help short-term and long-term investors or policy makers have more information in their decision making.

2. Research method and research data

Basing on papers of Gradojevic (2013), Chan et al. (2008), Ciner (2011), Ozer & Kamisli (2016), this study estimates the United States, Japanese and Vietnamese stock market index volatilities by GARCH model (Bollerslev 1986), tests volatility spillover by Granger Causality Test (Granger 1969), and analyze these relations in frequency domain by the approach proposed by Breitung & Candelon (2006).

2.1. Data

Daily data of Standard and Poor 500 (S&P 500) Composite Index, Nikkei 225 and VN-Index (proxy for the United States, Japanese and Vietnamese stock indices) from January 01, 2012 to May 31, 2016 is collected. Each time series has 1,152 observations.

The return $r_t$ was calculated using the following formula:

$$ r_t = \ln \frac{P_t}{P_{t-1}} $$

where $P_t$ is the market index at the time $t$, $\ln(x)$ is natural logarithm of $x$. To be more specific, $P_t$ are the S&P 500, Nikkei 225 and VN-Index in this paper.

The returns are used to estimate the volatilities of both markets through GARCH model (Bollerslev 1986)

2.2. GARCH model

Assume that return $r_t$ follows a simple time series model such as a stationary ARMA($p,q$) model with some explanatory variables. In other words, we entertain the model (Tsay 2005)

$$ r_t = \phi_0 + \sum_{i=1}^{k} \beta_i x_{it} + \sum_{i=1}^{p} \phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i} + a_t $$

for $r_t$, where $k, p, \text{ and } q$ are non-negative integers, and $x_{it}$ are explanatory variables, $a_t$ are error terms of model.

The first model that provides a systematic framework for volatility modeling is the ARCH model by Robert F. Engle (1982). Bollerslev (1986) proposed a useful extension known as the generalized ARCH (GARCH) model as shown below:

$$ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 $$
where $\sigma_t^2$ is the conditional variance or volatility, $a_t = \sigma_t \varepsilon_t$ with $\{\varepsilon_t\}$ is a sequence of identically distributed random variables with mean of 0 and variance of 1.0, $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$. Here, it is understood that $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$. The latter constraint on $(\alpha_i + \beta_i)$ implies that the unconditional variance of $a_t$ is finite, whereas its conditional variance $\sigma_t^2$ evolves over time. $\varepsilon_t$ is often assumed to be a standard normal or standardized Student-t distribution or generalized error distribution. The $\alpha_i$ and $\beta_j$ are referred to as ARCH and GARCH parameters, respectively (Ruey S. Tsay 2005).

The volatilities of the United States, Japanese and Vietnamese stock market indices can be estimated through GARCH model and the volatility spillover can be tested by Granger Causality Test (Granger 1969).

2.3. **Granger Causality Test**

Granger (1969) proposed a method for testing the “causality” of time series, named the Granger Causality Test. It is a statistical hypothesis test for determining whether one time series is useful in forecasting another.

The Granger Causality Test observes whether $x$ causes $y$ by seeing how much of the current $y$ can be explained through the past values of $y$ and lagged values of $x$. $y$ is said to be Granger-caused by $x$ if $x$ helps in the prediction of $y$, or equivalently if the coefficients on the lagged $x$’s are statistically significant.

The Granger Causality test can be done by using VAR (Vector Autoregression) as below: (Gujarati 2004)

\[
\begin{align*}
    y_t &= \alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_{l} y_{t-l} + \beta_1 x_{t-1} + \cdots + \beta_{l} x_{t-l} + \varepsilon_t \\
    x_t &= \alpha_0 + \alpha_1 x_{t-1} + \cdots + \alpha_{l} x_{t-l} + \beta_1 y_{t-1} + \cdots + \beta_{l} y_{t-l} + u_t
\end{align*}
\]

and test the null hypothesis:

\[\beta_1 = \beta_2 = \cdots = \beta_l = 0\]

for each equation. The null hypothesis is that $x$ does not Granger-cause $y$ in the first regression and that $y$ does not Granger-cause $x$ in the second regression.

The volatility spillover from the United States and Japanese to Vietnamese stock markets can be tested by applying the Granger Causality Test to volatility data of these markets.

2.4. **Testing for causality: A frequency-domain approach**

The frequency domain causality test developed by Breitung and Candelon (2006) is based on the framework of Geweke (1982) and Hosoya (1991). Let $z_t = [x_t, y_t]'$ be a two-dimensional time series vector with $t = 1, \ldots, T$. It is assumed that $z_t$ has a finite-order VAR representation:

\[\theta(L) z_t = \varepsilon_t\]
where $\Theta(L) = I - \theta_1 L - \cdots - \theta_1 L^p$ is a $2 \times 2$ lag polynomial with $L^k z_t = z_{t-k}$. It is assumed that the vector $\varepsilon_t$ is white noise with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t') = \Sigma$, where $\Sigma$ is a positive definite matrix. Next, let $G$ be the lower triangular matrix of the Cholesky decomposition $G'G = \Sigma^{-1}$, such that $E(\eta_t \eta_t') = I$ and $\eta_t = G\varepsilon_t$. The system is assumed to be stationary, implying the following MA representation:

$$z_t = \Phi(L)\varepsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$$= \Psi(L)\eta_t = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}$$

where $\Phi(L) = \Theta(L)^{-1}$ and $\Psi(L) = \Phi(L)G^{-1}$. Applying the Fourier transformation to this representation, the spectral density of $x_t$ can be expressed as:

$$f_x(\omega) = \frac{1}{2\pi} \left\{ \left| \Psi_{11}(e^{-i\omega}) \right|^2 + \left| \Psi_{12}(e^{-i\omega}) \right|^2 \right\}$$

The measure of causality suggested by Geweke (1982) and Hosoya (1991) is defined as:

$$M_{y \rightarrow x}(\omega) = \log \left[ \frac{2\pi f_x(\omega)}{\left| \Psi_{11}(e^{-i\omega}) \right|^2} \right]$$

$$= \log \left[ 1 + \left| \frac{\Psi_{12}(e^{-i\omega})}{\Psi_{11}(e^{-i\omega})} \right|^2 \right]$$

To test the hypothesis that $y$ does not cause $x$ at frequency $\omega$, the following null hypothesis is used:

$$M_{y \rightarrow x}(\omega) = 0$$

Breitung & Candelon (2006) shows that the null hypothesis $M_{y \rightarrow x}(\omega) = 0$ is equivalent to a linear restriction on the VAR coefficients. First, they use $\Psi(L) = \Theta(L)^{-1}G^{-1}$ and $\Psi_{12}(L) = -\frac{g^{22}\Theta_{12}(L)}{|\Theta(L)|}$ (where $g^{22}$ is the lower diagonal element of $G^{-1}$ and $|\Theta(L)|$ is the determinant of $\Theta(L)$) to express the null hypothesis as:

$$\left| \theta_{12}(e^{-i\omega}) \right| = \left| \sum_{k=1}^{p} \theta_{12,k}\cos(k\omega) - \sum_{k=1}^{p} \theta_{12,k}\sin(k\omega)i \right| = 0$$

where $\theta_{12,k}$ is the $(1,2)$-element of $\Theta_k$. Thus, a necessary and sufficient set of conditions for $|\theta_{12}(e^{-i\omega})| = 0$ is:

$$\sum_{k=1}^{p} \theta_{12,k}\cos(k\omega) = 0$$

$$\sum_{k=1}^{p} \theta_{12,k}\sin(k\omega) = 0$$
The notation can be simplified by letting $a_j = \theta_{11,j}$ and $\beta_j = \theta_{12,j}$. Then, the VAR equation for $x_t$ can be written as:

$$x_t = a_1x_{t-1} + \cdots + a_p x_{t-p} + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \varepsilon_{1t}$$

The hypothesis $M_{y \rightarrow x}(\omega) = 0$ is equivalent to the linear restriction:

$$H_0: R(\omega) \beta = 0$$

where $\beta = [\beta_1, \ldots, \beta_p]'$ and:

$$R(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \ldots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \ldots & \sin(p\omega) \end{bmatrix}$$

Like the conventional causality test, the Wald test statistic based on the above linear restriction is asymptotically distributed as $\chi^2(2)$ for $\omega \in (0, \pi)$ (Yanfeng 2013). As in Breitung & Candelon (2006), Gradojevic (2013) and Yanfeng (2013), to assess the statistical significance of the causal relationship between stock market volatilities, the causality measure for the frequency $\omega$ is compared to the 5% critical value of a $\chi^2$-distribution with 2 degrees of freedom (5.99).

3. Results

3.1. Descriptive statistical

Table 1 provides some descriptive statistical properties of daily market returns for the three countries.

**Table 1: Descriptive Statistics of Daily Returns of the United States, Japanese and Vietnamese Stock Indices**

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500 (U.S)</th>
<th>Nikkei 225 (Japan)</th>
<th>VN-Index (Vietnam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000444</td>
<td>0.000619</td>
<td>0.000491</td>
</tr>
<tr>
<td>Median</td>
<td>0.000182</td>
<td>0.000089</td>
<td>0.000138</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.008092</td>
<td>0.013869</td>
<td>0.010925</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.238174</td>
<td>-0.235324</td>
<td>-0.555895</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.821383</td>
<td>6.267740</td>
<td>5.784339</td>
</tr>
</tbody>
</table>

*Source: author’s calculation*

As Table 1 indicates, the means of the return of all market indices are positive, however, these values are small, which are consistent with the reality of the United States, Japanese and Vietnamese stock markets at this time because they were tested in the recovery period from global financial crisis of 2008. The negative skewness value of the studied cases implies that this is an asymmetric distribution that skews to the left. The kurtosis in all markets is larger than the three, implying a peaked distribution in comparison with the normal distribution

3.2. GARCH models
Autocorrelation functions of the return and the Augmented Dickey-Fuller (ADF) are employed to test the stationarity of time series. The results indicate that they are stationary. Moreover, autocorrelation functions and partial autocorrelation functions of the squared return contend a strong autocorrelation in the time series; that is, the time series are dependent, which indicates the existence of the GARCH effect.

The authors employ the Box-Jenkins methodology (Gujarati, 2004) to identify maximum orders \( p \) and \( q \) in ARMA \((p,q)\) of the mean equation of returns and GARCH \((1,1)\) to describe the volatility. This study uses shorter time series not long time series such as several decades old daily data or hourly data from a single year, thus the use of GARCH \((1,1)\) instead of GARCH \((m,s)\) with \( m \) and \( s \) being larger than one (Engle, 2001).

The GARCH \((1,1)\) model is taken into account and evaluated with the support of ARMA \((p,q)\) in mean equation to confirm the correspondent models which can ensure the statistical significance of coefficients and eliminate GARCH effects. Then, the Akaike Info Criterion (AIC) is used to choose an optimal model. After many trials, the best models for the markets are found and shown in Table 2.

**Table 2: GARCH estimates in the United States, Japanese and Vietnamese stock markets**

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500</th>
<th>Nikkei 225</th>
<th>VN-Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Equation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varnothing_0 )</td>
<td>0.000599***</td>
<td>0.000768***</td>
<td>0.000570**</td>
</tr>
<tr>
<td></td>
<td>(0.000107)</td>
<td>(0.000271)</td>
<td>(0.000272)</td>
</tr>
<tr>
<td>( \varnothing_1 )</td>
<td>0.119620***</td>
<td>-0.084045***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.043029)</td>
<td>(0.014341)</td>
<td></td>
</tr>
<tr>
<td>( \varnothing_2 )</td>
<td>0.113164**</td>
<td>0.311741***</td>
<td>0.067967**</td>
</tr>
<tr>
<td></td>
<td>(0.048681)</td>
<td>(0.015406)</td>
<td>(0.026717)</td>
</tr>
<tr>
<td>( \varnothing_3 )</td>
<td>-0.329775***</td>
<td></td>
<td>0.540978***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.054493)</td>
</tr>
<tr>
<td>( \varnothing_4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varnothing_5 )</td>
<td>-0.277089***</td>
<td>0.936879***</td>
<td>-0.086530***</td>
</tr>
<tr>
<td></td>
<td>(0.035025)</td>
<td>(0.009158)</td>
<td>(0.031128)</td>
</tr>
<tr>
<td>( \varnothing_6 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>0.779344***</td>
<td>0.338641***</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.046483)</td>
<td>(0.052317)</td>
<td></td>
</tr>
<tr>
<td>θ₂</td>
<td>-0.148784***</td>
<td>0.071137***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.040740)</td>
<td>(0.009394)</td>
<td></td>
</tr>
<tr>
<td>θ₃</td>
<td>-0.128651***</td>
<td>-0.309458***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042889)</td>
<td>(0.010000)</td>
<td></td>
</tr>
<tr>
<td>θ₄</td>
<td>0.349014***</td>
<td>-0.534386***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006785)</td>
<td>(0.057689)</td>
<td></td>
</tr>
<tr>
<td>θ₅</td>
<td>-0.20625***</td>
<td>-0.104646***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007037)</td>
<td>(0.032699)</td>
<td></td>
</tr>
<tr>
<td>θ₆</td>
<td>0.225452***</td>
<td>-0.966563***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028471)</td>
<td>(0.007488)</td>
<td></td>
</tr>
<tr>
<td>θ₇</td>
<td>0.347644***</td>
<td>-0.368964***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.043682)</td>
<td>(0.056450)</td>
<td></td>
</tr>
<tr>
<td>θ₈</td>
<td>-0.059629**</td>
<td>-0.000006***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027958)</td>
<td>(0.000002)</td>
<td></td>
</tr>
<tr>
<td>θ₉</td>
<td>-0.000006***</td>
<td>-0.000002***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028471)</td>
<td>(0.007488)</td>
<td></td>
</tr>
</tbody>
</table>

Variance Equation

| α₀   | 0.166897***  | 0.203472*** |
|      | (0.027420)  | (0.014773)  |
| β₁   | 0.734633*** | 0.618950*** |
|      | (0.041581)  | (0.018579)  |

Notes: *, **, and *** respectively denote the statistical significance at 10%, 5% and 1%.
Source: Author’s calculation

The coefficients α (ARCH parameter) and β (GARCH parameter) for the comprehensive indices of all studied markets are statistically significant at 1%. This
implies that the volatility of all markets indices depends on both volatility and errors (representing unexpected upheavals) of the previous periods. In other words, the volatility in all markets is affected by the unexpected upheaval of return, and volatility of previous periods (Nghi, 2012). Based on the models above, the volatilities of the three markets indices are estimated. These volatilities are used to test volatility spillover through the Granger Causality Test (Granger, 1969).

3.3. Volatility Spillover

The estimated volatility data of the United States, Japanese and Vietnamese stock indices are tested for stationarity with the ADF (Augmented Dickey-Fuller) test. The results are in Table 3

Table 3: ADF test for volatilities of the United States and Vietnamese volatilities

<table>
<thead>
<tr>
<th>Hypothesis H₀</th>
<th>S&amp;P500</th>
<th>Nikkei 225</th>
<th>VN-Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-Statistic</td>
<td>-7.879497</td>
<td>-5.088506</td>
<td>-10.60252</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Reject at 1% significance level</td>
<td>Reject at 1% significance level</td>
<td>Reject at 1% significance level</td>
</tr>
</tbody>
</table>

Source: Author’s calculation

All volatility time series are stationary. Thus, it is suitable to apply the Granger Causality Test to these time series to test the volatility spillover from the United States and Japanese to Vietnamese markets. Using the Akaike Info Criterion (AIC), VAR(3) models were selected for both cases. The Granger Causality Test results with three lag orders are displayed in Table 4.

Table 4: The volatility spillover from the United States and Japanese to Vietnamese Stock Markets

<table>
<thead>
<tr>
<th>Hypothesis H₀</th>
<th>Granger Causality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volatility in the United States stock market does not spill over volatility in the Vietnamese stock market</td>
</tr>
<tr>
<td>F-Statistic</td>
<td>4.79704</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Rejected at the 1% significance level</td>
</tr>
</tbody>
</table>

Source: Author’s calculation

The results in Table 5 show that there is a significant volatility spillover from the United States to the Vietnamese stock markets. These results are consistent with
reality because the United States is the world’s largest national economy and it can affect other countries including Vietnam. This is also an evidence of Vietnam’s integration into the world economy. The size of the Vietnamese capital market and market openness are increasing, helping the Vietnamese stock market to link to the world financial markets. The results support the conclusion of Tsutsui & Hirayama (2005) that most, if not all, of the literature offers evidence on the existence of the stock market linkage.

The results also show that the United States market volatility is one of determinants that can forecast Vietnamese market volatility. These findings have very important implications for both investors and policy makers. First, because the United States stock market can spillover to Vietnamese stock market, keeping up to date with the United States stock exchange and its fluctuations plays an important role in investing in Vietnamese stock market. Secondly, the results also suggest that Vietnam’s policy makers need to follow the United States market volatility to manage risks in the Vietnamese stock market. If an increase of the volatility of the United States stock market occurs, Vietnam’s policy makers should have the suitable remedies to preclude the panic among investors and reduce the market volatility in Vietnam. Price limit policy could be used in this case. This tool was used by the State Security Commission of Vietnam (SSC) in 2008 to reduce the effect of global financial crisis to Vietnam’s stock market. Moreover, as suggested by Nghi (2012), government should manipulate better market information and assure the adequacy and transparency of the market. Lastly, because of the significant relationship between the United States and Vietnam stock market, the strategy of the United States investors investing stocks in Vietnam to reduce the diversifiable risk is not effective. Thus, investors from the United States should seek other markets to diversify their investment portfolios.

The results in Table 5 also indicate that there is no evidence of volatility spillover from the Japanese to Vietnamese stock market. It suggests that investors from Japan could diversify their portfolios by investing the Vietnamese stock market.

3.4. Causality in frequency domain

Since Granger causality tests are providing only one statistic for the whole sample, it is not well suited to distinguish short and long run effects (Ozer and Kamisli, 2016). Because the causality results can be different between frequency bands (Granger and Lin, 1995), the traditional Granger causality tests are unable to examine these. To obtain a more precise analysis, a causality test in frequency domain by Breitung & Candelon (2006) should be used.

Frequency domain causality test (Breitung and Candelon, 2006) is applied to markets’ volatility data to test the volatility spillover from the United States and Japanese to Vietnamese stock markets at different significance levels. Using the
Akaike Info Criterion (AIC), VAR(3) models were selected for both cases. The test results with ten different frequencies between 0 and \( \pi \) can be seen in Table 5 and Table 6.

**Table 5: Volatility Spillover from the United States to the Vietnamese Stock Markets in frequency domain**

<table>
<thead>
<tr>
<th>Frequency ( \omega )</th>
<th>Test Statistic ( \chi^2 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010000</td>
<td>12.284</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>0.35667</td>
<td>12.109</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>0.70333</td>
<td>12.415</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>1.0500</td>
<td>13.489</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>1.3967</td>
<td>14.023</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>1.7433</td>
<td>14.212</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>2.0900</td>
<td>14.285</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>2.4367</td>
<td>14.317</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>2.7833</td>
<td>14.331</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>3.1300</td>
<td>14.335</td>
<td>Reject ( H_0 )</td>
</tr>
</tbody>
</table>

*Source: Author’s calculation*

**Table 6: Volatility Spillover from the Japanese to the Vietnamese Stock Markets in frequency domain**

<table>
<thead>
<tr>
<th>Frequency ( \omega )</th>
<th>Test Statistic ( \chi^2 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010000</td>
<td>3.826</td>
<td>Do Not Reject ( H_0 )</td>
</tr>
<tr>
<td>0.35667</td>
<td>3.938</td>
<td>Do Not Reject ( H_0 )</td>
</tr>
<tr>
<td>0.70333</td>
<td>3.986</td>
<td>Do Not Reject ( H_0 )</td>
</tr>
<tr>
<td>1.0500</td>
<td>3.863</td>
<td>Do Not Reject ( H_0 )</td>
</tr>
</tbody>
</table>

*Source: Author’s calculation*
The results in Table 5 show that there is a significant volatility spillover from the United States to Vietnamese stock market at all frequencies. These results are consistent with the results of the volatility spillover testing using traditional Granger Causality Test in Table 4.

Moreover, the results in Table 5 show that the statistic test values ($\chi^2$-distribution values) are not the same at different frequencies. Example, for $\omega = 0.35667$, corresponding to the approximate 18-day cycle (the formula for cycle: $T = \frac{2\pi}{\omega}$), the statistic test value is 12.109; but for $\omega = 2.7833$; corresponding to the approximate 2-day cycle, the statistic test value is 14.331. These results support the hypothesis that causality may be different for each frequency (Granger and Lin, 1995). In more detail, assume that the volatility spillover from the United States to Vietnamese stock markets is considered at the 0.2% significance level, the critical value of $\chi^2$-distribution is 12.43. Consequently, the null hypothesis is not rejected for $\omega = 0.35667$, but is rejected for $\omega = 2.7833$, i.e., there is a significant volatility spillover from the United States to Vietnamese stock market at $\omega = 2.7833$, but not at $\omega = 0.35667$. It implies that the investors should have different decision-making styles based on their investment cycles.

However, in spite of the differences, the test statistic values only change in range $[10.847, 13.007]$ making $H_0$ (i.e., the hypothesis stating that the volatility in the United States stock market does not cause the volatility in Vietnamese stock market) be rejected at all frequencies at 5% significance level. In other words, there is a significant spillover from the United States to the Vietnamese market at all cycles. Hence, both short- and long-term investors must follow the information and fluctuations in the United States stock market when investing on Vietnamese stock market.

The evidence of the hypothesis that causality can be different for each frequency (Granger and Lin, 1995) is clearer in the results of testing the volatility spillover from the Japanese to the Vietnamese stock market in Table 6. At 10% significance level, the results are consistent with the results of the volatility spillover testing using traditional

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Test Statistic</th>
<th>Decision</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35667</td>
<td>12.109</td>
<td>Do Not Reject $H_0$</td>
<td>Do Not Reject $H_0$</td>
</tr>
<tr>
<td>2.7833</td>
<td>14.331</td>
<td>Do Not Reject $H_0$</td>
<td>Do Not Reject $H_0$</td>
</tr>
</tbody>
</table>

Source: Author’s calculation
Granger Causality Test in Table 4. However, if the 15% significance level is considered, the results of the two approaches are different. Applying traditional Granger Causality Test, the p-value is 0.26. Therefore, the null hypothesis is not rejected at the 15% significance level. However, the results are changed when the frequency domain approach is applied. The results in Table 6 show that at the 15% significance level, the null hypothesis is rejected at frequencies that less than or equal to 1.3967, corresponding to the approximate 5-day cycle. However, the null hypothesis is not rejected at frequencies that greater than or equal to 1.7433, corresponding to the approximate 4-day cycle. Therefore, at the 15% significance level, there is a significant volatility spillover from the Japanese to the Vietnamese stock markets at $\omega = 1.3967$, but not at $\omega = 1.7433$. These results support the hypothesis that causality can be different for each frequency (Granger and Lin, 1995).

The $\chi^2$ statistics of 100 frequencies $\omega \in (0, \pi)$ along with the 15% critical value (3.79) of the volatility spillover from the Japanese to the Vietnamese stock market test is presented in Figure 1.

**Figure 1: $\chi^2$ statistics for the volatility spillover from the Japanese to the Vietnamese stock market (at the 15% significance level)**
Note: the values of the $\chi^2$ test statistic are given by +. The 15% critical value (3.79) is given by solid line.

Source: Author's calculation

Figure 1 shows that at the 15% significance level, the null hypothesis of no causality is rejected for $\omega \leq 1.3967$, i.e., for cycles greater than or equal to 5 days, but it is not rejected for $\omega > 1.3967$, i.e., for cycles less than 5 days. The conclusion is that changes of the Nikkei 225 volatility affect the VN-Index volatility at cycles greater than or equal to 5 days. Therefore, at the 15% significance level, there is a significant volatility spillover from the Japanese to the Vietnamese stock market in the long run. In other words, there is a weak volatility spillover from the Japanese to Vietnamese in the long run. Hence, long-term investors (cycles greater than or equal to 5 days) should follow the information and fluctuations in the Japanese stock market when investing on Vietnamese stock market, but it is not necessary for the short-term ones (cycles less than 5 days). The results also suggest that policy makers in Vietnam should pay more attention to the long-term effects on Japan's economy.

There are a number of possible causes of volatility spillover between the United States and Japanese to Vietnam stock markets. First, both the United States and Japan are large economies and they can affect other countries, including Vietnam. This is supported by empirical results of Ng (2000), Ko and Lee (1991), Salma (2015), Mohammadi and Tan (2015). Secondly, common global shocks may be affecting firms across borders and make all markets fluctuate together. Thirdly, large change in the stock price index of one country usually receives a great deal of attention from investors of other countries like a “sun-spot” (Tsutsui and Hirayama, 2005). In this case, investors from Vietnam tend to update the fluctuation in the United States and Japanese stock markets for their investment decisions. For this point of views, the weaker the spillover from the Japanese to Vietnam market implies that investors from Vietnam focus more on the fluctuation in the United States market. It explains why there is no spillover from the Japanese to Vietnamese market in short term, i.e., although there is a weak relationship between two markets, investors from Vietnam do not react immediately from Japanese volatilities. Lastly, portfolios of institutional investors are significantly diversified across national boundaries, and international capital movements caused by portfolio adjustments of these investors affect stock prices worldwide (Nishimura, Tsutsui, and Hirayama, 2015). The increasing investment of foreign investors in HOSE is an evidence for this explanation.
In summary, the Vietnamese stock volatility is affected by the United States stock volatility in both long-term and short-term, but not by the Japanese volatility at the 5% significance level. However, at 15% significance level, VN-Index volatility is affected by Nikkei 225 volatility in long term, but not for short term. Therefore, investors and policy makers in Vietnam should follow both the United States and long-term Japanese volatility for their decision making.

4. Conclusions

In this paper, daily returns of S&P 500, Nikkei225, and VN-Index from January 01, 2012 to May 31, 2016 were used to test for the volatility spillover from the United States and Japanese Stock Exchange to the Vietnamese Stock Exchange. The GARCH model proposed by Bollerslev (1986) is used to estimate volatility; Granger Causality Test (Granger 1969) is applied to test for the presence of volatility spillover between markets; and Spectral Granger Causality Test by Breitung & Candelon (2006) is used to test causality at specific frequencies.

The results show that there is a significant volatility spillover from the United States to the Vietnamese stock market, but there is no evidence of a volatility spillover from the Japanese to the Vietnamese stock market (at 5% significance level). That is the United States market influence to Vietnamese market. These findings have very important implications for both investors and policy makers. First, the investors should follow the information and fluctuations in the United States stock exchange when investing in Vietnam. Secondly, investors from the United States should find other markets for their portfolio diversification strategies. Finally, Vietnam policy makers need to keep in touch to the United States market volatility to handle Vietnamese stock market risks.

Moreover, the results of spectral volatility spillover test show that statistic test values ($\chi^2$- distribution values) are different for each frequency. Applying frequency domain approach, the results show that although there is no evidence of volatility spillover from the Japanese to the Vietnamese stock market at 5% significance level, VN-Index volatility is affected by Nikkei 225 volatility in the long run at 15% significance level. It suggests that investors and policy makers in Vietnam should pay more attention to the long-term effects on Japan's economy.

Therefore, these results provide evidence that the linkages between stock market volatilities may vary across frequency spectrum bands. Thus, because time domain causality test may fail to fully capture such links, frequency domain analysis should be applied to have a deep insight into volatility spillover between stock markets. This method will help short- and long-term investors have more information in investment decisions based on their needs.
In summary, this paper has two main contributions. First, this study provides evidence of a significant volatility spillover from the United States to the Vietnamese market, and a weak volatility spillover from the Japanese to the Vietnamese in the long term, then suggests important implications for both investors and policymakers. Secondly, it shows that the volatility spillover between two stock markets changes at different frequencies. To our best knowledge, this nonlinear relationship between stock market volatilities is not mentioned in previous literature. Then, frequency domain approach should be used in testing the volatility spillover between stock exchanges.

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