Blind Separation Using Second Order Statistics for Non-stationary Signals

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Abstract. In the signal processing area, blind source separation (BSS) is a method aiming to recover independent sources from their linear instantaneous mixtures without resorting to any prior knowledge, such as mixing matrices and sources. There have been increased attentions given to blind source separation in many areas, including wireless communication, biomedical imaging processing, multi-microphone array processing, and so on in recent years. In this paper, we propose a new simple BSS technique that exploits second order statistics for non-stationary sources. Our technique utilizes the algebraic structure of the signal model and the subspace structures so as to efficiently recover sources with interference of noise. Computer simulations have demonstrated that in comparison with other existing methods, our method has better performance in the regimes of low and medium SNRs. For high SNRs, our method is not as promising methods such as the method called AC ("alternating columns")-DC ("diagonal centers") algorithm, but it gives reasonable performance.

Keywords: blind source separation, signal model, computer simulation, signal to noise ratio.

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1. Introduction

In recent years, blind source separation (BSS) has received much attention in such applications as multi-path channel identification and equalization, direction of arrival (DOA) estimation in sensor arrays, speech enhancement with multiple microphones, biomedical signal analysis, and crosstalk removal in multi-channel communications. This wide applicability is due to the fact that BSS can be used to process data from multi-sensor measurements with no need to model the underlying physical phenomena accurately.

Most of BSS algorithms can be classified into two types according to how the source sequences are modeled. The first type [1, 2] deals with stationary signals, such as modulated digital signals in communication system, while the second [3-10] involves non-stationary signals, such as speech signals and video signals. In this thesis, we consider non-stationary source sequences. Many BSS algorithms for non-stationary signals have been proposed in the last decade. Because of the non-stationarity of the source signals, most BSS algorithms attempt to get the estimation of the mixing matrix or its inverse by simultaneously diagonalizing all covariance matrices. For instance, Parra et al. [6] proposed a method that minimizes the error between the covariance matrices, which are obtained from observed signals, and the iterative estimated covariance matrices by the gradient descent method to estimate the mixing matrix or its inverse.

In this thesis, we proposed a new algorithm which utilizes the algebraic structure of the signal model to separate source signals from observed mixtures by alternating projection and to suppress the noise effect by subspace projection. Some simulations are performed to show that the proposed method has better performance in comparison to other existing methods in cases where mixtures are perturbed by significant noise. The remaining parts of the paper are organized as follows. In Section 2, we present the problem statement and some general assumptions of our problem. In Section 3, the new proposed algorithm is presented explicitly. We first derive the identifiability of the proposed algorithm, and then present the channel identifiability, reconstruction of sources, and noise reduction in order. Then some simulation results are provided to justify its performance in Section 4. Finally some conclusions and discussions are drawn in Section 5.

2. Problem Statement and Assumptions

The following notations will be used throughout the development that follows.

<table>
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<th>Notation</th>
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<tr>
<td></td>
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<tr>
<td>E{ }</td>
<td>The expectation operator</td>
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<tr>
<td>⊗</td>
<td>Kronecker product</td>
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<tr>
<td>vec{ }</td>
<td>A vector which is $[a_{i_1}, \ldots , a_{p_1}, a_{i_2}, \ldots , a_{p_2}, \ldots , a_{i_q}, \ldots , a_{p_q}]^T$</td>
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for given matrix \( A = [a_{ij}] \in \mathbb{R}^{m \times q} \)

\[
\text{diag}\{x\} = \text{a diagonal matrix whose } i\text{-th element is } x_i \text{ where } i = 1, \cdots, N \text{ for } x \in \mathbb{R}^N
\]

\[
\text{mat}\{y\} = \text{a matrix whose } ij\text{-th element is } y_{(i-1)N+j} \text{ where } i, j = 1, \cdots, N \text{ for } y \in \mathbb{R}^{N^2}
\]

Superscript “T” Transpose of vectors or matrices

In the simplest form of blind source separation, the \( N \)-dimensional vector sequences \( \{x(t)\} \) of observed signals are modeled as a linear mixture of \( K \) sequences of source signals with the \( N \times K \) mixing matrix \( H \), given by

\[
x(t) = Hs(t) + v(t)
\]

where \( v(t) \) is \( N \times 1 \) noise vector. The illustration of the model (1) is shown in the figure 1 as follows:

![Fig. 1. Signal model](image)

The objective is to reconstruct the sources \( s(t) \) from the observations \( x(t) \) up to a fixed permutation and some scaling factors. Next, let us make some general assumptions as follows:

(A1) Source signals are zero-mean and statistically independent of each other.

(A2) \( H \in \mathbb{R}^{N \times K} \) is of full column rank. The matrix \( H \) can be expressed as \( [h_1, \cdots, h_K] \), where \( h_k \) is the k-th column of \( H \).

(A3) \( N \geq K \).

(A4) Noise is zero-mean spatially uncorrelated noise with \( R_v \equiv \mathbb{E} \{v(t)v^T(t)\} = \sigma_v^2 I \), and is statistically independent of the source signals.
3. The Proposed Algorithm Based on Alternating Projection

3.1. Blind identifiability

First, we divide source signal vector into M frames in time domain, and each frame has L samples. For sufficiently small frame length, we can further assume that the source signal are “Quasi-Stationary,” that is, the source signals are wide-sense stationary within each frames. We herein refer the length of each frame to as “the stationary time.” Let $R_{m}$ and $S_{m}$ denote, respectively, the autocorrelation matrix of the observed signals and the autocorrelation matrix of the source signals at time frame $m$. The autocorrelation matrix of the source signals in $m$-th frame is written as

$$R_{s,m} = E\{s(t)s^T(t)\} = \begin{bmatrix} d_{m1} & 0 & \cdots & 0 \\ 0 & d_{m2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{mk} \end{bmatrix}$$

for $t \in [(m-1)L+1,mL]$ where $d_{mk}$ is the average power of the $k$-th source within the $m$-th frame.

Using (A1), and (A2), the noise-free model (1) leads to the autocorrelation matrix

$$R_{s,m} = H R_{s,m} H^T = \sum_{k=1}^{K} d_{mk} h_k h_k^T$$

where $R_{s,m}$ by assumption is diagonal for all $m$, and where $d_{mk}$ is the $k$-th diagonal value of $R_{s,m}$. Defining $y_{m} \triangleq \text{vec} \{ R_{s,m} \}$ and using the property $\text{vec} \{ EF \} = (G^T \otimes E) \text{vec} \{ F \}$, we have

$$y_{m} = \sum_{k=1}^{K} d_{mk} (h_k \otimes h_k)$$

Stacking all $y_{m}$ as the matrix $Y = [y_1, \cdots, y_M]$, yields

$$Y = AD^T$$

where $A = [a_1, \cdots, a_K] = [h_1 \otimes h_1, \cdots, h_K \otimes h_K]$ , $D = [d_1, \cdots, d_M]$ and $d_i = [d_{ik}, \cdots, d_{ik}]$ which represents the vector resulted from the variation of average power of the $k$-th source. Non-stationary signals, such as speech signals, usually exhibit significant variations in power for each frame. Therefore, we can reasonably assume that the power variations with
time of each source are linearly independent, that is, $D$ is of full column rank. And then we have the following two lemmas.

**Lemma 1.** If $H$ is of full column rank then $A$ is also of full column rank.

**Lemma 2.** If $D$ is of full column rank, then $\text{range}(Y) = \text{range}(A)$.

From lemma 1 and lemma 2, the dimension of range space of $Y$ is $K$, and $\{a_1, \cdots, a_K\}$ is not only a basis of range space of $Y$ but also a basis whose each vector, $a_k$, can be decomposed as the form of $h_k \otimes h_k$. For convenience, we define a vector $x \in \mathbb{R}^{N}$ to be Kronecker decomposable if it can be expressed as $x = f \otimes f$ for some $f \in \mathbb{R}^{N}$. Obviously, a vector, $\alpha a_k$ for $k = 1, \cdots, K$ and for $\alpha > 0 \in \mathbb{R}$, is still Kronecker decomposable and are in range($Y$). To see whether there exists other Kronecker decomposable vector except $\{\alpha a_k | k = 1, \cdots, K \text{ and } \alpha > 0 \in \mathbb{R}\}$ in range($Y$), we suppose that a vector, $a \in \text{range}(Y)$, satisfies

$$a = h \otimes h, \text{ for some } h \in \mathbb{R}^{N} \quad (6)$$

Then we have

$$h \otimes h = \sum_{k=1}^{K} c_k a_k = \sum_{k=1}^{K} c_k h_k \otimes h_k \quad (7)$$

for some $c \in \mathbb{R}^K$ since $a \in \text{range}(Y)$. From (7),

$$\text{mat}(h \otimes h) = hh^T = \sum_{k=1}^{K} c_k h_k h_k^T = H \text{diag}([c]) H^T \quad (8)$$

where $H = [h_1, \cdots, h_K]$. Equation (8) implies that $H \text{diag}([c]) H^T$ is a rank one matrix. Such a condition holds only when $c$ contains one nonzero element since $H$ is of full rank. Thus,

$$hh^T = c_k h_k h_k^T \quad (9)$$

for some $k \in \{1, \cdots, K\}$. Note that $c_k > 0$ because $hh^T$ and $h_k h_k^T$ are positive definite matrices. Let $\alpha = c_k$ then equation (9) becomes

$$a = \alpha a_k \quad (10)$$

or

$$h = \pm \sqrt{\alpha} h_k \quad (11)$$
So far we have shown that \( \{ \alpha a_k \mid k = 1, \ldots, K \text{ and } \alpha > 0 \in \mathbb{R} \} \) is a unique set which consists of all Kronecker decomposable vectors in range\( \{ Y \} \). Next, let us summarize the consequence as following theorem:

**Theorem 1.** Let \( a = h \otimes h, h \in \mathbb{R}^K \), and \( P_a \) is the orthogonal projector of span\( \{ a_1, \ldots, a_K \} \) where \( a_k = h_k \otimes h_k, \forall k = 1, \ldots, K \). The condition

\[
P_a a = a
\]

is satisfied if and only if \( a = \alpha a_k \text{ for any } k = 1, \ldots, K \) and for any \( \alpha > 0 \in \mathbb{R} \). The above theorem tells us that there is a unique set of Kronecker decomposable bases which is constructed from the mixing matrix \( H \) and lies in the subspace range\( \{ Y \} \). Thus, it provides the identifiability of the mixing matrix. The main idea of the proposed algorithm is that we estimate the mixing matrix \( H \) by finding out Kronecker decomposable vectors in range\( \{ Y \} \).

### 3.2. Channel Identification

In this section, we focus on the estimation of the mixing matrix \( H \) as the first step of our algorithm. The main objective of the proposed algorithm is to estimate \( H \), up to a permutation and scaling factor. We assume without loss of generality that \( \| h_k \|_2 = 1 \forall k = 1, \ldots, K \).

Define two sets as follows:

\[
\Omega_1 = \{ a \in \mathbb{R}^N \mid a = h \otimes h, h \in \mathbb{R}^K, \| h \|_2 = 1 \}
\]

(13)

\[
\Omega_2 = \text{span}\{ y_1, \ldots, y_K \} = \text{span}\{ a_1, \ldots, a_K \}
\]

(14)

By Theorem 1, the intersection of these two sets is:

\[
\Omega = \Omega_1 \cap \Omega_2 = \{ a_1, \ldots, a_K \}
\]

(15)

Now, let us focus on how to find the element in the intersection \( \Omega \). Define the projector function \( P_1 : \mathbb{R}^N \rightarrow \Omega_1 \) as

\[
P_1(u) = \arg \min_{a \in \Omega_1} \| u - a \|_2
\]

(16)

and the projector function \( P_2 : \mathbb{R}^N \rightarrow \Omega_2 \) as

\[
P_2(v) = \arg \min_{a \in \Omega_2} \| v - a \|_2
\]

(17)

The optimization problem (16) can be rewritten as
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\[
\min_{\mathbf{v} \in \Omega} \| \mathbf{v} - \mathbf{a} \|^2 \tag{18}
\]

Using (13) and (18), we have

\[
\min_{\mathbf{h} \in \Omega} \| \mathbf{h}^T \mathbf{u} - \mathbf{h}^T \|^2 \tag{19}
\]

which equals to

\[
\min(\text{trace}(\mathbf{u}^T \mathbf{v} \mathbf{u}^T) - 2\mathbf{h}^T \mathbf{u} \mathbf{v} + (\mathbf{h}^T \mathbf{v})^2) \tag{20}
\]

Discarding the constant terms yields

\[
\max_{\| \mathbf{h} \|=1} \mathbf{h}^T \mathbf{u} \mathbf{v} \tag{21}
\]

Obviously, the solution is the unit eigenvector of \( \mathbf{u} \) corresponding to the largest eigenvalue. As to equation (17), it is a classic least square problem whose solution is the projection of \( \mathbf{v} \) onto \( \text{span}(\mathbf{y}_1, \ldots, \mathbf{y}_M) \). Consequently, the solutions to (16) and (17) are

\[
P_1(\mathbf{u}) = s_{\text{max}}(\mathbf{u}) \otimes u_{\text{max}}(\mathbf{u}) \tag{22}
\]

\[
P_2(\mathbf{v}) = \mathbf{v} \tag{23}
\]

where \( s_{\text{max}}(\mathbf{u}) \) is the dominant unit eigenvector of \( \mathbf{u} \) and \( P_i \) is the orthogonal projector of \( \Omega_i \) which can be obtained from left singular matrix by truncated singular value decomposition (SVD) of the matrix \( \mathbf{Y} \). In other words, letting \( \mathbf{Y} = \mathbf{U}_K \Sigma K \mathbf{V}_K^T \) be the truncated SVD of \( \mathbf{Y} \) corresponding to the K largest singular values, we have \( P_i = \mathbf{U}_K \mathbf{U}_K^T \). The algorithm starts with a randomly generated iterate \( \mathbf{u}_0 \in \mathbb{R}^M \), and then it alternatively projects the iterates onto \( \Omega_1 \) and \( \Omega_2 \) with the projector functions \( P_1 \) and \( P_2 \) until it converges. Once it converges, we get one normalized column of mixing matrix. The proposed alternating projection algorithm for finding one channel column is summarized as follows:

- \( \hat{\mathbf{h}} = \text{ALT} \mathbf{PRJ}(\mathbf{u}_k) \)
- Input initial vector \( \mathbf{u}_0 \in \mathbb{R}^M \), and set \( k = 1, l = 0 \).

-repeat

\[
l = l + 1
\]

\[
\mathbf{v}_l = P_2(\mathbf{u}_{l-1})
\]

\[
\mathbf{u}_l = P_1(\mathbf{v}_l)
\]

until \( \| \mathbf{u}_l - \mathbf{v}_l \| \leq \varepsilon \)
Output \( \hat{h} \) = the dominant unit eigenvector of \( \text{mat}\{u_i\} \).

Next, let us consider the problem about how to find out all columns of the mixing matrix. Intuitively, we can repeatedly perform ALTPRJ with randomly generated initial vectors \( u_0 \) until \( K \) linearly independent vectors are obtained. However, the number of times of performing ALTPRJ may be large in order to obtain all columns of the mixing matrix. To avoid finding too many repeated columns, we proposed a heuristic method to efficiently get the rest of the columns. First, we find the first column \( \hat{h}_1 \) by ALTPRJ with randomly generated vector \( u_0 \in \mathbb{R}^{N} \). Next, ALTPRJ is repeatedly performed with initial vectors projected onto different subspaces. Supposing that we have estimated \( k-1 \) columns, \( \hat{h}_1, \cdots, \hat{h}_{k-1} \), we let \( u_k^{(1)} = (P_{\hat{h}_1} \perp P_{\hat{h}_2} \perp \cdots \perp P_{\hat{h}_{k-1}}) u \) where \( u \in \mathbb{R}^{N} \) is generated randomly and where \( P_{\hat{h}_j} \) is the orthogonal complement projector of \( \text{span}\{ \hat{h}_1, \cdots, \hat{h}_{j-1} \} \). With the initial vectors projected onto the subspaces corresponding to the columns we have estimated, the algorithm is repeatedly performed until all columns are estimated. By simulation, we find that the choice of initial vector greatly affects the result of searching the channel columns. With the choice of this initial vector, the algorithm can estimate mixing matrix efficiently.

Finally, the reconstructed source signals can be obtained from the observed signals multiplied by the pseudo inverse of the estimated mixing matrix, \( \hat{H}^\dagger \). The following is the proposed procedure for blind source separation based on the alternating projection:

- **Given** \( Y, \varepsilon \) and let \( P = U U^\top \).
- **\( \hat{h}_1 \) = ALTPRJ\( (u_0^{(1)}) \) where \( u_0^{(1)} \in \mathbb{R}^{N} \) is generated randomly.
- for \( k = 2 \) to \( K \)
  - \( \hat{H}_k = [\hat{h}_1, \cdots, \hat{h}_{k-1}] \) and \( P_{\hat{h}_k} = I - \hat{H}_k (\hat{H}_k^\top \hat{H}_k)^{-1} \hat{H}_k^\top \)
  - repeat
    - \( u_k^{(1)} = (P_{\hat{h}_1} \perp P_{\hat{h}_2} \perp \cdots \perp P_{\hat{h}_{k-1}}) u \) where \( u \in \mathbb{R}^{N} \) is generated randomly.
    - \( \hat{h}_k = \text{ALTPRJ}(u_k^{(1)}) \)
    - until \( |\hat{h}_k^\top \hat{h}_j| \leq \varepsilon \) for all \( j = 1, \cdots, k-1 \)
  - endfor
- **Output** \( \hat{H} = [\hat{h}_1, \cdots, \hat{h}_K] \)
- **\( \hat{s}[t] = \hat{H}^\top x[t] \)**

### 3.3. Noise Reduction

When noise is present, the covariance matrix of the observed signals based
on the signal model (1.1) becomes
\[
\hat{R}_{r,m} = HR_{r,m}H^T + R_s
= HR_{r,m}H^T + \sigma^2 I
\]
(24)

In the case for \( N > K \), the eigen decomposition of \( \hat{R}_{r,m} \) has the form
\[
U \begin{bmatrix}
\sigma_1^2 + \sigma^2 \\
\vdots \\
\sigma_K^2 + \sigma^2 \\
\sigma^2 \\
\vdots \\
\sigma^2 \\
\end{bmatrix} U^T
\]
(25)
where \( U \) is the matrix of eigenvectors of \( \hat{R}_{r,m} \) and \( \{\sigma_1^2, \cdots, \sigma_K^2\} \) are eigenvalues of \( HR_{r,m}H^T \). Thus, we can estimate the noise variance, \( \sigma^2 \), by calculating the average of the \( K + 1 \)-th, \( \ldots \), \( N \)-th smallest eigenvalues. With the estimated noise variance, we have the noise-free covariance matrix
\[
HR_{r,m}H^T = \hat{R}_{r,m} + \hat{\sigma}^2 I
\]
(26)

But in the case for \( N = K \), the noise variance cannot be estimated because \( HR_{r,m}H^T \) and \( \hat{R}_{r,m} \) have the same rank which is \( N \).

For this problem, we proposed a noise reduction method which is based on subspace projection. Defining \( \tilde{y}_m \triangleq \text{vec}(R_{r,m}) \) and stacking all \( \tilde{y}_m \) as the matrix \( \tilde{Y} = [\tilde{y}_1, \cdots, \tilde{y}_M] \), yields
\[
\tilde{Y} = AD^T + \sigma^2 \text{vec}(I)I^T
\]
(27)
where \( I \) is a \( N \times N \) identity matrix and where \( 1 \) is a \( M \times 1 \) vector whose elements are all ones. Observing equation (28), we can rewrite it as
\[
\tilde{Y} = \begin{bmatrix}
d_1^T \\
\vdots \\
d_K^T \\
d_1^T \\
\end{bmatrix}
\]
(28)

Apparently, the row space of \( \tilde{Y} \) is spanned by \( \{d_1^T, \cdots, d_K^T, I^T\} \) and the noise component only lies on the DC direction (the direction of \( I^T \)). In general, we can reasonably assume that are linearly independent of \( I^T \) because of significant power variations of the source signals. Thus, expecting to cancel the component of the noise, we project \( \tilde{Y} \) onto the complement space of
4. Simulation Results

In this section, simulation results are presented for speech sources. We compare the performance of mixing matrix estimation with some existed algorithms under the case that noise is added on the observation signals. Three (8 kHz, 12 seconds) are used in the simulations. The number of sensors is N = 3, and the length of each frames, T, is 320 samples (0.04 second).

4.1. Experiment 1

In this experiment, we investigate the noise effect upon the performance of the algorithms. The definition of signal to noise ratio (SNR) is as follows:

$$\text{SNR} = 10 \log_{10} \frac{E\{\|x[t]-v[t]\|^2\}}{E\{\|v[t]\|^2\}}$$  \hspace{1cm} (30)

Because some of the algorithms estimate the mixing matrix and the others estimate the inverse matrix of that, we define two performance indexes to measure the performance of source separation for fair comparison as follows:

$$\Phi = 10 \log_{10} E\{\min_{\Pi} \|H - \hat{H}\Pi\|_F^2\}$$

where H and \( \hat{H} \) are, respectively, the mixing matrix and the estimated one, and where \( \Pi \in \mathbb{R}^{k \times k} \) is permutation matrix. To ignore the scaling ambiguity, we restrict each column of H to be of unit norm. The expectation is with respect to the trial, and in this example 1000 Monte Carlo runs were used to evaluate that expected value. This measurement tells us how completely a BSS algorithm recovers sources with ignoring permutation and scaling ambiguity. The advantage of this measurement is that its calculation can be transformed into a convex optimization problem so that it can be done efficiently. These two performance indexes can be calculated by modifying the method of calculating Wasserstein distance proposed in [11]. We compared
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The proposed method with Gradient Gescent Method (LUCAS) [6], Gaussian Likelihood Maximization (PHAM) [7], ACDC [12], and QDIAG [13] in this example. The simulation result is shown in Fig. 2. From the figure, we found that our algorithm has the best performance when SNR < 20dB and converges to a floor, $\Phi_{ch} = -33$ dB, very quickly. Though the performance of our algorithm is worse than the performance of QDIAG, ACDC, and PHAM when SNR > 20dB, the performance which our algorithm converges at is still good in practice. To see this, we sum all the matrices $\hat{H}^{-1}H$, which are permuted and normalized artificially to eliminate the permutation ambiguity and the scaling ambiguity so that the diagonal elements of $\hat{H}^{-1}H$ are ones, of each independent runs and average them for fixed SNR=50dB, then we have

$$\frac{1}{M_c} \sum_{i=1}^{M_c} \hat{H}^{-1}H = \begin{bmatrix} 1 & 0.0031 & 0.0025 \\ -0.0045 & 1 & 0.0003 \\ 0.0006 & -0.0075 & 1 \end{bmatrix}$$ (31)

where $i$ denotes the index of independent runs and the number of independent runs, $M_c$, is 1000. This means that the diagonalization of $\Phi_{ch} = -33$ dB is good enough to separate sources.

![Fig. 2. Performance ($\Phi_{ch}$) of blind channel identification under different SNR](image-url)
4.2. Experiment 2

In this experiment, we apply our method to recover real speech sources from linearly mixed signals with noise perturbation. SNR is set to be 20dB. The waveforms of original sources, linearly mixtures, and recovered sources are respectively shown in Fig 3. The results indicate that our method can well separate sources from mixtures which is perturbed by noise.

Fig 3.1. The waveforms of the sources
Fig 3.2. The waveforms of the linearly mixed signals
Fig 3.3. The waveforms of the recovered signals.
5. Conclusion

In this thesis, we have presented a new blind source separation algorithm for non-stationary signals using second order statistics. The proposed algorithm was derived from a subspace perspective and may perfectly identify the channel even in the presence of noise. Through computer simulations, we have demonstrated that the proposed algorithm outperformed the existing algorithms when the effect of noise is conspicuous. However, there is a performance limit when the noise is small for our algorithm. It would be an interesting future direction to improve our algorithm for small noise cases.

6. References

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